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# Methodology of gross output growth decomposition in KLEMS productivity accounts for the Polish economy

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The commonly practiced decomposition of gross value added growth into the contributions of production factors (labour and capital services) and multifactor productivity (MFP) can be developed into a decomposition of gross output growth, on the condition, however, that price indices for intermediate consumption are available. This opens the possibility to clear the residually calculated MFP from sometimes substantial effects of substitution between the production factors and the intermediate consumption. If the data on gross output and intermediate consumption are of good quality, and the tool effects associated with additional computations are negligible, then this additional procedure of gross output growth decomposition can bring important analytical benefits, associated with outsourcing (i.e. the main mechanism of labour substitution by intermediate consumption) monitoring and associated with the blurred boundary between capital investments and intermediate consumption outlays (i.e. in the area of changing accountancy and tax regulations and more or less strict ad hoc activity of the revenue administration, and also some other circumstances).

In processing the appropriate calculations, it is best to remain consistent with the already carried out computations for the gross value added growth decomposition.

The starting point is the formula for the gross output growth, at the given aggregation level $j$ in period $t$[1]:

$$\Delta \ln Y_{jt} = \bar{v}_{jt}^{X} \Delta \ln X_{jt} + \bar{v}_{jt}^{K} \Delta \ln K_{jt} + \bar{v}_{jt}^{L} \Delta \ln L_{jt} + \Delta \ln A_{jt}^{Y} \quad (1)$$

[1] See: references to the parallel document: *Methodology of gross value added growth decomposition in KLEMS productivity accounts for the Polish economy*.

where $Y$ is gross output, $X$ – intermediate consumption, $K$ – capital, $L$ – labour (the contributions of these two factors understood as contributions of their services), whereas $A^Y$ is the so-called *multifactor productivity* (MFP), that can be considered as a variant of TFP. These values are subscribed to indicate that they concern industries $j$ and periods $t$. $\Delta$, for all values under this symbol, denotes changes between periods $t$ and $t-1$, usually considered as yearly periods. $\bar{v}$ with appropriate superscripts and subscripts denote average value shares of the given factors in gross output (indicated in superscripts as $X$, $K$ or $L$) between periods $t$ and $t-1$, that are calculated according to formula $\bar{v} = \left(v_t + v_{(t-1)}\right)/2$ (for simplicity the subscript $j$ present in formula (1) has been omitted here).

The above-mentioned formula should be made consistent with the formula for the decomposition of gross value added growth:

$$\Delta \ln V_{jt} = \bar{w}_{jt}^K \Delta \ln K_{jt} + \bar{w}_{jt}^L \Delta \ln L_{jt} + \Delta \ln A_{jt}^V \tag{2}$$

where $V$ stands for gross value added, and the other symbols (with appropriate superscripts and subscripts) have the same meaning as in formula (1) but, with the exception of capital $K$ and labour $L$, take different values. The analogical average shares $\bar{w}$ (in GVA) are not identical to average shares $\bar{v}$ from formula (1) (they are expressed in percentages and calculated in a similar way to average shares $\bar{v}$ by linear interpolation). Also, the contribution of MFP in the decomposition of gross value added $V$ growth is not identical in percentage points to the contribution of MFP in the decomposition of gross output $Y$ growth, although its absolute growth in the ideal case where there were no variability associated with intermediate consumption should be identical.

The consistency between formulae (1) and (2) will be achieved if some terms from formula (2) are inserted in formula (1):

$$\Delta \ln Y_{jt} = \bar{v}_{jt}^X \Delta \ln X_{jt} + \overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^K \Delta \ln K_{jt} + \overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^L \Delta \ln L_{jt} + \Delta \ln A_{jt}^Y \tag{3}$$

In formula (3) the contributions of production factors (labour and capital) to gross value added growth from formula (2) are multiplied by the ratio between gross value added and gross output value at industry $j$ level. These ratios are calculated (by similarity to the shares) with the use of linear interpolation, as arithmetic averages between two periods.

The factor contributions from formula (3) are decomposed in KLEMS accounting into sub-contributions:

$$\overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^K \Delta \ln K_{jt} = \overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^{KIT} \Delta \ln KIT_{jt} + \overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^{KNIT} \Delta \ln KNIT_{jt} \tag{4}$$

$$\overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^L \Delta \ln L_{jt} = \overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^L \Delta \ln H_{jt} + \overline{\left(\frac{V_{jt}}{Y_{jt}}\right)} \bar{w}_{jt}^L \Delta \ln LC_{jt} \tag{5}$$

Where $KIT$ indicates ICT capital and $KNIT$ – non-ICT capital, and where $H$ indicates hours worked and $LC$ – labour composition, but there exist other variants of labour factor decompositions[2].

In order to make the contribution of MFP to gross output growth comparable with the contribution of MFP to gross value added growth it is required to transform it according to the following formulae:

$$\Delta \, ln \, A_{jt}^{V*} = \overline{\left(\frac{Y_{Jt}}{V_{Jt}}\right)} \Delta \, ln \, A_{jt}^{Y} \tag{6}$$

The values in the parentheses are arithmetic averages between two periods ratios between gross output and gross value added (i.e. conversely in comparison with the previous formulae (3), (4) and (5)). The results should approximately meet the condition: $\Delta \, ln \, A_{jt}^{V*} \approx \Delta \, ln \, A_{jt}^{V}$ (the term on the right-hand side is taken from formula (2)). If it is not so, then it can be assessed that the substitution between the production factors and the intermediate consumption is substantial, i.e. important changes are under way in the outsourcing and in the way of attributing some outlays to either capital outlays or intermediate consumption outlays. It is therefore possible to monitor these processes from the macroeconomic point of view. Lastly, a sub-decomposition of the intermediate consumption contribution into sub-contributions of energy, materials and services would allow even further analyses of these processes.

In the same way as for the gross value added growth basic decomposition, the data for the gross output growth decomposition are presented as contributions to aggregate gross output growth (Excel table marked by A, B, C and D) or as contributions to industry gross output growths (Excel tables marked by A', B', C' and D').

[2] A more detailed description of this issue is delivered in the document attached in parallel on the internet site, entitled: *Methodology of developed decomposition of labour factor contribution in KLEMS productivity accounts for the Polish economy.*