STATISTICS IN TRANSITION new series, September 2019
Vol. 20, No. 3, pp. 81–95, DOI 10.21307/stattrans-2019-025
Submitted – 07.04.2018; Paper ready for publication – 28.09.2018

ESTIMATION OF PRODUCT OF TWO POPULATION MEANS BY MULTI-AUXILIARY CHARACTERS UNDER DOUBLE SAMPLING THE NON-RESPONDENTS

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ABSTRACT

This paper considers the problem of estimating the product of two population means using the information on multi-auxiliary characters with double sampling the non-respondents. Classes of estimators are proposed for estimating P under two different situations [discussed by Rao (1986, 90)] using known population mean of multi-auxiliary characters. Further, this problem has been extended to the case when population means of the auxiliary characters are unknown and they are estimated on the basis of a larger first phase sample. In this situation, a class of two phase sampling estimators for estimating P is suggested using multi-auxiliary characters with unknown population means in the presence of non-response. The expressions of bias and mean square error of all the proposed estimators are derived and their properties are studied. An empirical study using real data sets is given to justify the theoretical considerations.

Key words: product, bias, mean square error, auxiliary characters, non-response.

1. Introduction

While conducting sample surveys in the field of agriculture, socio-economic and forest research, one may be interested in the estimation of the product of two population means. For example- if we want to estimate the total population of persons in a District using villages as the sampling units, then we will estimate the product of average number of occupied houses in a village and average number of persons in a house in that village and the population of the District can be obtained by multiplying the estimate of product to the number of villages. The auxiliary characters used in this circumstance may be the area, the number of cultivators, the number of agricultural labours, etc., of the village. Similarly one may use the amount of manure, water and seeds supplied to each plot as the auxiliary characters in the estimation of total yield of a crop within an agricultural field, which can be obtained by estimating the product of average area per plot

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and the average yield per plot and then multiplying this estimate of product by the number of plots in the agricultural field.

Many authors like Singh (1965, 67, 69), Shah & Shah (1978), Singh (1982a,b), Ray and Singh (1985), Khare (1987), Srivastativa *et al.* (1988), Khare (1990, 91) considered the problem of estimating the ratio and the product of two population means and suggested estimators/classes of estimators using the information of auxiliary character(s).

When the population means of the auxiliary characters are not known then in this case Singh (1982c) proposed generalized double sampling estimators for the ratio and product of population parameters using double sampling scheme. Further, Khare (1992) proposed the class of estimators for the product of two population means using multi-auxiliary characters with known and unknown population means.

In some cases, it happens that the information on the main characters and auxiliary characters may not be available in practice due to the occurrence of non-response for the selected units in the sample viz. while conducting the yield of a crop in an agricultural field, it may be possible that the data on yield of a crop as well as the area of the plot may be made available for some selected plots for which the information on the auxiliary characters may or may not be known due to lack of reporting the owner of the field. To reduce the effect of non-response in the estimation of parameter for a variable, Hansen and Hurwitz (1946) suggested a method of sub-sampling on the non-responding units and proposed an unbiased estimator for population mean. Further, following Hansen and Hurwitz (1946) strategies of sub-sampling the non-responding units, Khare and Sinha (2002 a, b, 2004 a, b, 2007, 2012 a, b), Singh *et al.* (2007), Singh and Kumar (2008) proposed some classes of estimators for the ratio/product of two population means using auxiliary character(s) in the presence of non-response. The objective of this paper is to suggest classes of estimators for estimating the product of two population means using information available on multi-auxiliary characters in the presence of non-response under different situations and study their theoretical and empirical properties.

2. Proposed classes of estimators

Let $y_i (i = 1, 2)$ and $x_j (j = 1,2, \dots, p)$ be the main and auxiliary characters under study having non-negative k^{th} value $Y_{ik}, X_{jk}; (k = 1,2, \dots, N)$ with population means $\bar{Y}_i\ (i = 1, 2)$ of study characters and $\bar{X}_j\ (j = 1, 2, \dots \dots, p)$ of auxiliary characters respectively. The whole population is supposed to be divided into two non-overlapping unknown strata of N_1 responding and N_2 non-responding units such that $N_1 + N_2 = N$. Let n be the size of the sample drawn from the population of size N using simple random sampling without replacement (SRSWOR) scheme of sampling and it has been observed that n_1 units respond and n_2 units do not respond in the sample of size n. We have considered that the responding and non-responding units are same for both the study and auxiliary characters. The stratum weights of responding and non-responding groups are given by $W_1 = N_1/N$ and $W_2 = N_2/N$ and their estimates are respectively given by $\widehat{W}_1 = w_1 = n_1/n$ and $\widehat{W}_2 = w_2 = n_2/n$. Further, from the non-responding units n_2, we draw a subsample of size $m(= n_2\,\delta^{-1}, \delta > 1)$ using SRSWOR technique of sampling and collect the information by the direct interview for y_i $(i = 1, 2)$. Using the methodology of Hansen and Hurwitz (1946), the unbiased estimator for $\bar{Y}_i$ $(i = 1, 2)$ based on the information of $(n_1 + m)$ units is given by

$$\bar{y}_{i(HH)} = w_1 \bar{y}_{i1} + w_2 \bar{y}^*_{i2}, \quad i = 1, 2 \tag{1}$$

where $\bar{y}_{i1}$ and $\bar{y}^*_{i2}$ are the sample means of y_i based on n_1 and m units respectively.

The estimator $\bar{y}_{i(HH)}$ is unbiased and has variance given by

$$V\left(\bar{y}_{i(HH)}\right) = V^{(i)} = \lambda\, S^2_{y_i} + \lambda_\delta\, S^2_{y_{i(2)}}, \tag{2}$$

where $S^2_{y_i}$ and $S^2_{y_{i(2)}}$ are the population mean square of y_i for the entire and non-responding group of the population and $\lambda = \frac{N-n}{Nn}$, $\lambda_\delta = \frac{N_2}{N\,n}(\delta - 1)$.

Similarly, the estimator $\bar{x}_{j(HH)}$ $(j = 1,2, \dots, p)$ for the population mean $\bar{X}_j$ $(j = 1,2, \dots, p)$ is given by

$$\bar{x}_{j(HH)} = w_1 \bar{x}_{j1} + w_2 \bar{x}^*_{j2}, \quad j = 1, 2, \dots., p \tag{3}$$

where $\bar{x}_{j1}$ and $\bar{x}^*_{j2}$; $(j = 1,2 \dots, p)$ are the sample means of the character x_j $(j = 1,2, \dots, p)$ based on n_1 and m units respectively.

Let $\hat{P}(= \prod_{i=1}^{2} \bar{y}_{i(HH)})$ denote conventional estimator for the product of two population means $(P = \bar{Y}_1 . \bar{Y}_2)$ in the presence of non-response on study characters. Utilizing the information of auxiliary characters with known population means, the two different proposed classes of estimators for P under two different cases discussed by {Rao (1986), page 220} are as follows:

- **For the first case**, it is assumed that the population mean $\bar{X}_j$ $(j = 1,2, \dots, p)$ is known, and incomplete information occurred on y_i $(i = 1, 2)$ and x_j $(j = 1,2, \dots, p)$ for the selected units in the sample of size n. In such situation, we propose a class of estimators t_p for the product of two population means (P) using multi-auxiliary characters $x_j (j = 1,2, \dots, p)$ with known population means as:

$$t_p = g\left(\prod_{i=1}^{2} \bar{y}_{i(HH)},\ (u_1, u_2, \dots, u_p)'\right) = g(\varphi, \boldsymbol{u}'), \tag{4}$$

such that

$$g(P, \boldsymbol{e}') = P, \qquad g_1(P, \boldsymbol{e}') = \left(\frac{\partial}{\partial \varphi} g(\varphi,\ \boldsymbol{u}')\right)_{(P,\ \boldsymbol{e}')} = 1 \tag{5}$$

where $\boldsymbol{u}$ and **e** denote the column vectors $(u_1, u_2, \dots, u_p)'$ and $(1,1, \dots, 1)'$ respectively. We also denote $\varphi = \prod_{i=1}^{2} \bar{y}_{i(HH)}$ and $u_j = \bar{x}_{j(HH)} / \bar{X}_j$; $j = 1,2, \dots, p$.

- **For the second case**, it is assumed that the population mean $\bar{X}_j$ $(j = 1,2, \dots, p)$ is known, and incomplete information occurred on y_i $(i = 1, 2)$ only but complete information on x_j $(j = 1,2, \dots, p)$ for the selected units in the sample of size n. In this situation, we propose a class of estimators t^*_p for the product of two population means (P) using multi-auxiliary characters $x_j (j = 1,2, \dots, p)$ with known population means as:

$$t_p^* = h\left(\prod_{i=1}^{2} \bar{y}_{i(HH)},\ (\omega_1, \omega_2, \dots, \omega_p)'\right) = h(\varphi, \boldsymbol{\omega}'), \tag{6}$$

such that

$$h(P, \boldsymbol{e}') = P, \qquad h_1(P, e') = \left(\frac{\partial}{\partial \varphi} h(\varphi,\ \boldsymbol{\omega}')\right)_{(P,\ \boldsymbol{e}')} = 1 \tag{7}$$

where $\boldsymbol{\omega}$ denotes the column vectors $(\omega_1, \omega_2, \dots, \omega_p)'$ and $\omega_j = \bar{x}_j / \bar{X}_j$; $j = 1,2, \dots, p$.

For the expansion of the functions $g(\varphi, \boldsymbol{u}')$ and $h(\varphi, \boldsymbol{\omega}')$, it is supposed that whatever be the sample chosen for any sampling design, $(\varphi, \boldsymbol{u}')$ [or $(\varphi, \boldsymbol{\omega}')$] assumes a value in a bounded closed convex subset D_p [or D_p^*] of the $(p + 1)$ dimensional real space containing the point $(P,\ \boldsymbol{e}')$. In D_p [or D_p^*], the function $g(\varphi, \boldsymbol{u}')$ [or $h(\varphi, \boldsymbol{\omega}')$] is continuous and bounded. The first and second partial derivatives of $g(\varphi, \boldsymbol{u}')$ [or $h(\varphi, \boldsymbol{\omega}')$] exist and are continuous and bounded in D_p [or D_p^*].

Here, $g_1(\varphi, \boldsymbol{u}')$ [or $h_1(\varphi, \boldsymbol{\omega}')$] and $g_2(\varphi, \boldsymbol{u}')$ [or $h_2(\varphi, \boldsymbol{\omega}')$] denote the first partial derivatives of $g(\varphi, \boldsymbol{u}')$ [or $h(\varphi, \boldsymbol{\omega}')$] with respect to φ and $\boldsymbol{u}'$ [or $\boldsymbol{\omega}'$] respectively. The second partial derivatives of $g(\varphi, \boldsymbol{u}')$, $h(\varphi, \boldsymbol{\omega}')$ with respect to $\boldsymbol{u}'$ and $\boldsymbol{\omega}'$ are respectively denoted by $g_{22}(\varphi, \boldsymbol{u}')$, $h_{22}(\varphi, \boldsymbol{\omega}')$] and first partial derivative of $g_2(\varphi, \boldsymbol{u}')$ [or $h_2(\varphi, \boldsymbol{\omega}')$] with respect to φ is denoted by $g_{12}(\varphi, \boldsymbol{u}')$ [or $h_{12}(\varphi, \boldsymbol{\omega}')$].

Under the regularity conditions imposed on $g(\varphi, \boldsymbol{u}')$ and $h(\varphi, \boldsymbol{\omega}')$, it may be seen that the bias and mean square error of the estimators t_p and t_p^* will always exist.

In order to derive the expressions for bias and mean square error of the estimators under large sample approximation, let us assume

$$\epsilon_{0i} = \frac{\bar{y}_i^* - \bar{Y}_i}{\bar{Y}_i},\ \ \epsilon_j = \frac{\bar{x}_j^* - \bar{X}_i}{\bar{X}_i},\ \epsilon_j' = \frac{\bar{x}_j - \bar{X}_j}{\bar{X}_j},\ \text{with}\ \ E(\epsilon_{0i}) = E(\epsilon_j) = E(\epsilon_j') = 0$$

and $|\epsilon_{0i}| < 1$, $|\epsilon_j| < 1$, $|\ \epsilon_j'| < 1\ \ \forall\ i = 1, 2; j = 1, 2, \dots \dots, p.$

Now, using SRSWOR method of sampling, we have

$$E(\epsilon_{0i}^2) = \frac{V(\bar{y}_{i(HH)})}{\bar{Y}_i^2} = \frac{1}{\bar{Y}_i^2}\left[\lambda\, S_{y_i}^2 + \lambda_\delta\, S_{y_{i(2)}}^2\right], \quad E(\epsilon_j^2) = \frac{V(\bar{x}_{j(HH)})}{\bar{X}_j^2} = \frac{1}{\bar{X}_j^2}\left[\lambda\, S_{x_j}^2 + \lambda_\delta\, S_{x_{j(2)}}^2\right],$$

$$E(\epsilon_j'^2) = \frac{V(\bar{x}_j)}{\bar{X}_j^2} = \lambda \frac{S_{x_j}^2}{\bar{X}_j^2},\ \ E(\epsilon_{01}, \epsilon_{02}) = \frac{Cov(\bar{y}_{1(HH)}, \bar{y}_{2(HH)})}{\bar{Y}_1 . \bar{Y}_2} = \frac{1}{\bar{Y}_1 . \bar{Y}_2}\left[\lambda\, S_{y_1 y_2} + \lambda_\delta S_{y_1 y_2 (2)}\right],$$

$$E(\epsilon_{0i}, \epsilon_j) = \frac{Cov(\bar{y}_{i(HH)}, \bar{x}_{j(HH)})}{\bar{Y}_i . \bar{X}_j} = \frac{1}{\bar{Y}_i . \bar{X}_j}\left[\lambda\, S_{y_i x_j} + \lambda_\delta S_{y_i x_{j(2)}}\right],$$

$$E(\epsilon_{0i}, \epsilon_j') = \frac{Cov(\bar{y}_{i(HH)}, \bar{x}_j)}{\bar{Y}_i . \bar{X}_j} = \lambda \frac{S_{y_i x_j}}{\bar{Y}_i . \bar{X}_j}, \qquad E\left(\epsilon_j', \epsilon_{j'}'\right) = \frac{Cov(\bar{x}_j, \bar{x}_{j'})}{\bar{X}_j . \bar{X}_{j'}} = \lambda \frac{S_{x_j x_{j'}}}{\bar{X}_j . \bar{X}_{j'}},$$

$$E(\epsilon_j, \epsilon_{j'}) = \frac{Cov(\bar{x}_{j(HH)}, \bar{x}_{j'(HH)})}{\bar{X}_j . \bar{X}_{j'}} = \frac{1}{\bar{X}_j . \bar{X}_{j'}}\left[\lambda\, S_{x_j x_{j'}} + \lambda_\delta S_{x_j x_{j'} (2)}\right]. \tag{8}$$

The contribution of the terms involving the powers in ϵ_{0i}, ϵ_j and ϵ_j' of order higher than two in the bias and mean square error is assumed to be negligible.

3. Bias and mean square error (MSE) of t_p and t_p^*

Expanding the functions $g(\varphi, \boldsymbol{u}')$ about the point $(P, \boldsymbol{e}')$ using Taylor's series up to the second order partial derivative, we get

$$t_p = g(P, \boldsymbol{e}') + (\varphi - P)g_1(P, \boldsymbol{e}') + (\boldsymbol{u} - \boldsymbol{e})' g_2(P, \boldsymbol{e}') + \frac{1}{2}[(\varphi - P)^2 g_{11}(\varphi^*, \boldsymbol{u}^{*\prime}) + 2(\varphi - P)(\boldsymbol{u} - \boldsymbol{e})' g_{12}(\varphi^*, \boldsymbol{u}^{*\prime}) + (\boldsymbol{u} - \boldsymbol{e})' g_{22}(\varphi^*, \boldsymbol{u}^{*\prime})(\varphi - P)]$$

Using condition given in equation (5), we get

$$t_p = P + (\varphi - P) + (\boldsymbol{u} - \boldsymbol{e})' g_2(P, \boldsymbol{e}') + \frac{1}{2}[(\varphi - P)^2 g_{11}(\varphi^*, \boldsymbol{u}^{*\prime}) + 2(\varphi - P)(\boldsymbol{u} - \boldsymbol{e})' g_{12}(\varphi^*, \boldsymbol{u}^{*\prime}) + (\boldsymbol{u} - \boldsymbol{e})' g_{22}(\varphi^*, \boldsymbol{u}^{*\prime})(\boldsymbol{u} - \boldsymbol{e})].$$

Similarly, expanding the $h(\varphi, \boldsymbol{\omega}')$ about the point $(P, \boldsymbol{e}')$ and using equation (7), we get

$$t_p^* = P + (\varphi - P) + (\boldsymbol{\omega} - \boldsymbol{e})' h_2(P, \boldsymbol{e}') + \frac{1}{2}\begin{bmatrix}(\varphi - P)^2 h_{11}(\varphi, \boldsymbol{\omega}') + 2(\varphi - P)(\boldsymbol{\omega} - \boldsymbol{e})' h_{12}\left(\varphi^*, \boldsymbol{\omega}^{*\prime}\right) \\ +(\boldsymbol{\omega} - \boldsymbol{e})' h_{22}(\varphi^*, \boldsymbol{\omega}^{*\prime})(\boldsymbol{\omega} - \boldsymbol{e})\end{bmatrix} \quad (9)$$

The expressions for bias and mean square error of t_p and t_p^* for any sampling design up to the terms of order n^{-1} are given by

$$Bias(t_p) = Bias\,(\varphi) + E(\varphi - P)(\boldsymbol{u} - \boldsymbol{e})' g_{12}\left(\varphi^*, \boldsymbol{u}^{*\prime}\right) + \frac{1}{2}E(\boldsymbol{u} - \boldsymbol{e})' g_{22}(\varphi^*, \boldsymbol{u}^{*\prime})(\boldsymbol{u} - \boldsymbol{e}), \quad (10)$$

$$MSE(t_p) = MSE\,(\varphi) + 2E(\varphi - P)(\boldsymbol{u} - \boldsymbol{e})' g_2(P, \boldsymbol{e}') + E\left(g_2(P, \boldsymbol{e}')\right)'(\boldsymbol{u} - \boldsymbol{e})(\boldsymbol{u} - \boldsymbol{e})'\, g_2(P, \boldsymbol{e}') \quad (11)$$

$$Bias(t_p^*) = Bias\,(\varphi) + E(\varphi - P)(\boldsymbol{\omega} - \boldsymbol{e})' h_{12}\left(\varphi^*, \boldsymbol{\omega}^{*\prime}\right) + \frac{1}{2}E(\boldsymbol{\omega} - \boldsymbol{e})' h_{22}(\varphi^*, \boldsymbol{\omega}^{*\prime})(\boldsymbol{\omega} - \boldsymbol{e}), \quad (12)$$

$$MSE(t_p^*) = MSE\,(\varphi) + 2E(\varphi - P)(\boldsymbol{\omega} - \boldsymbol{e})' h_2(P, \boldsymbol{e}') + E\left(h_2(P, \boldsymbol{e}')\right)'(\boldsymbol{\omega} - \boldsymbol{e})(\boldsymbol{\omega} - \boldsymbol{e})'\, h_2(P, \boldsymbol{e}'), \quad (13)$$

where $\varphi^* = P + \theta_P(\varphi - P)$, $\boldsymbol{u}^* = \boldsymbol{e} + \emptyset_{\mathbf{1}}(\boldsymbol{u} - \boldsymbol{e})$ and $\boldsymbol{\omega}^* = \boldsymbol{e} + \emptyset_{\mathbf{2}}(\boldsymbol{\omega} - \boldsymbol{e})$, such that $0 < \theta_P, \emptyset_{1j}$, $\emptyset_{2j} < 1$ and $\emptyset_{\mathbf{1}}$ and $\emptyset_{\mathbf{2}}$ are $(p \times p)$ diagonal matrix with j^{th} diagonal elements $\emptyset_{1j}$ and $\emptyset_{2j}$ respectively.

Differentiating equations (11) and (13) partially with respect to $g_2(P, \boldsymbol{e}')$ and $h_2(P, \boldsymbol{e}')$ respectively and equating them to zero, we get the conditions for the minimum value of the mean square error of t_p and t_p^*

$$g_2(P, \boldsymbol{e}') = -[E(\varphi - P)(\boldsymbol{u} - \boldsymbol{e})' / E(\boldsymbol{u} - \boldsymbol{e})(\boldsymbol{u} - \boldsymbol{e})'] \quad (14)$$

and

$$h_2(P, \boldsymbol{e}') = -[E(\varphi - P)(\boldsymbol{\omega} - \boldsymbol{e})' / E(\boldsymbol{\omega} - \boldsymbol{e})(\boldsymbol{\omega} - \boldsymbol{e})'] \quad (15)$$

respectively.

Now, putting the value of $g_2(P,\boldsymbol{e}')$ from equation (14) to (11) and $h_2(P,\boldsymbol{e}')$ from equation (15) to (13), the minimum value of the mean square error t_p and t_p^* will be given by

$$MSE(t_p)\big|_{min} = MSE\ (\varphi) - E(\varphi - P)(\boldsymbol{u}-\boldsymbol{e})'\ [E(\boldsymbol{u}-\boldsymbol{e})(\boldsymbol{u}-\boldsymbol{e})']^{-1}E(\varphi - P)(\boldsymbol{u}-\boldsymbol{e}) \quad (16)$$

$$MSE(t_p^*)\big|_{min} = MSE\ (\varphi) - E(\varphi - P)(\boldsymbol{\omega}-\boldsymbol{e})'\ [E(\boldsymbol{\omega}-\boldsymbol{e})(\boldsymbol{\omega}-\boldsymbol{e})']^{-1}E(\varphi - P)(\boldsymbol{\omega}-\boldsymbol{e}) \quad (17)$$

Considering SRSWOR method of sampling, let us define two $p \times p$ positive definite matrices $\boldsymbol{\mathcal{A}} = [\mathfrak{a}_{jj'}]$ and $\boldsymbol{\mathcal{A}_0} = [\mathfrak{a}_{0jj'}]$such that

$$\mathfrak{a}_{jj'} = \lambda\ \mathfrak{a}_{0jj'} + \lambda_\delta\ \mathfrak{a}_{0jj'(2)} \qquad \forall\ j \neq j' = 1,2,\ldots,p$$

where $\mathfrak{a}_{0jj'} = \rho_{x_j x_{j'}} C_{x_j} C_{x_{j'}}$, $\mathfrak{a}_{0jj'(2)} = \rho_{x_j x_{j'}(2)} C_{x_{j(2)}} C_{x_{j'(2)}}$, $C_{x_j}^2 = S_{x_j}^2/\bar{X}_i^2$, $C_{x_{j(2)}} = S_{x_{j(2)}}^2/\bar{X}_i^2$

$\rho_{x_j x_{j'}}$ - correlation coefficient between x_j and $x_{j'}$ for entire population,

$\rho_{x_j x_{j'}(2)}$ - correlation coefficient between x_j and $x_{j'}$ for non-responding group of population.

Then the expressions for bias and mean square error of t_p and t_p^* up to the terms of order (n^{-1}) in the case of SRSWOR method of sampling are given by

$$Bias(t_p) = Bias\ (\varphi) + P(\lambda\ \boldsymbol{\ell} + \lambda_\delta\ \boldsymbol{\ell}_{(2)})' g_{12}(\varphi^*, \boldsymbol{u}^{*\prime}) + \frac{1}{2} trace\ \boldsymbol{\mathcal{A}}\ g_{22}(\varphi^*, \boldsymbol{u}^{*\prime}), \quad (18)$$

$$MSE(t_p) = MSE\ (\varphi) + (g_2(P,\boldsymbol{e}'))'\ \boldsymbol{\mathcal{A}}\ g_2(P,\boldsymbol{e}') + 2\ P(\lambda\ \boldsymbol{\ell} + \lambda_\delta\ \boldsymbol{\ell}_{(2)})' g_2(P,\boldsymbol{e}'), \quad (19)$$

$$Bias(t_p^*) = Bias\ (\varphi) + \lambda\left\{P\boldsymbol{\ell}' h_{12}(\varphi^*, \boldsymbol{\omega}^{*\prime}) + \frac{1}{2} trace\ \boldsymbol{\mathcal{A}_0}\ h_{22}(\varphi^*, \boldsymbol{\omega}^{*\prime})\right\}, \quad (20)$$

and $$MSE(t_p^*) = MSE\ (\varphi) + \lambda\left\{(h_2(P,\boldsymbol{e}'))' \boldsymbol{\mathcal{A}_0}\ h_2(P,\boldsymbol{e}') + 2\ P\boldsymbol{\ell}'\ h_2(P,\boldsymbol{e}')\right\} \quad (21)$$

where $\boldsymbol{\ell} = (\ell_1, \ell_2, \ldots, \ell_p)'$ and $\boldsymbol{\ell}_{(2)} = (\ell_{1(2)}, \ell_{2(2)}, \ldots, \ell_{p(2)})'$ are two column vectors such that

$$\ell_j = \frac{S_{x_j}}{\bar{X}_j}\left\{\rho_{y_1 x_j}\frac{S_{y_1}}{\bar{Y}_1} + \rho_{y_2 x_j}\frac{S_{y_2}}{\bar{Y}_2}\right\} \text{ and } \ell_{j(2)} = \frac{S_{x_{j(2)}}}{\bar{X}_j}\left\{\rho_{y_1 x_{j(2)}}\frac{S_{y_1(2)}}{\bar{Y}_1} + \rho_{y_2 x_{j(2)}}\frac{S_{y_2(2)}}{\bar{Y}_2}\right\},$$

$$Bias\ (\varphi) = P\left\{\lambda\ \rho_{y_1 y_2}\frac{S_{y_1}}{\bar{Y}_1}\frac{S_{y_2}}{\bar{Y}_2} + \lambda_\delta\ \rho_{y_1 y_2(2)}\frac{S_{y_1(2)}}{\bar{Y}_1}\frac{S_{y_2(2)}}{\bar{Y}_2}\right\}, \quad (22)$$

$$MSE(\varphi) = P^2 \left\{\lambda\left(\frac{S_{y_1}^2}{\bar{Y}_1^2} + \frac{S_{y_2}^2}{\bar{Y}_2^2} + 2\rho_{y_1y_2}\frac{S_{y_1}}{\bar{Y}_1}\frac{S_{y_2}}{\bar{Y}_2}\right) + \lambda_\delta\left(\frac{S_{y_1(2)}^2}{\bar{Y}_1^2} + \frac{S_{y_2(2)}^2}{\bar{Y}_2^2} + 2\rho_{y_1y_2(2)}\frac{S_{y_1(2)}}{\bar{Y}_1}\frac{S_{y_2(2)}}{\bar{Y}_2}\right)\right\}, \quad (23)$$

$\rho_{y_1y_2}$- correlation coefficient between y_1 and y_2 for the entire population,
$\rho_{y_1x_j}$- correlation coefficient between y_1 and x_j for the entire population,
$\rho_{y_2x_j}$- correlation coefficient between y_2 and x_j for the entire population,
$\rho_{y_1y_{2(2)}}$- correlation coefficient between y_1 and y_2 for the non-responding group of population,
$\rho_{y_1x_{j(2)}}$- correlation coefficient between y_1 and x_j for the non-responding group of population,
$\rho_{y_2x_{j(2)}}$- correlation coefficient between y_2 and x_j for the non-responding group of population.

The conditions for which $MSE(t_p)$ and $MSE(t_p^*)$ will attain the minimum values are given by

$$g_2(P, \boldsymbol{e}') = -P(\lambda\,\boldsymbol{b} + \lambda_\delta\,\boldsymbol{b}_{(2)})\boldsymbol{\mathcal{A}}^{-1} \quad (24)$$

and $$h_2(P, \boldsymbol{e}') = -P\boldsymbol{b}\boldsymbol{\mathcal{A}}^{-1} \quad (25)$$

respectively. And, the values of the minimum mean square error for t_p and t_p^* are given by

$$MSE(t_p)\big|_{min} = MSE\ (\varphi) - P^2\left\{(\lambda\,\boldsymbol{b} + \lambda_\delta\,\boldsymbol{b}_{(2)})'.\boldsymbol{\mathcal{A}}^{-1}.(\lambda\,\boldsymbol{b} + \lambda_\delta\,\boldsymbol{b}_{(2)})\right\} \quad (26)$$

and $$MSE(t_p^*) = MSE\ (\varphi) - P^2\lambda\,\boldsymbol{b}'\boldsymbol{\mathcal{A}_0}^{-1}\,\boldsymbol{b}\ . \quad (27)$$

4. Extension of the proposed class of estimator to the case when population means of the auxiliary characters are unknown

In the case when the population means of the auxiliary characters $\bar{X}_j\ (j = 1,2, \dots, p)$ are unknown but sampling frame is available, we use two phase sampling technique to estimate the unknown population means of the auxiliary characters $x_1, x_2, \dots, x_p$. In two phase sampling scheme, we first select a larger sample of size n' from N using SRSWOR method of sampling and collect information regarding the auxiliary characters and estimate $\bar{X}_j\ (j = 1,2, \dots, p)$ based on n'units by $\bar{x}_j'\ (j = 1,2, \dots, p)$. Again, a second phase sample of size $n\ (< n')$ is drawn from n' units by SRSWOR method of sampling and observe the study characters $y_i (i = 1, 2)$. For the study characters $y_i (i = 1, 2)$, we observe that only n_1 units are responding and n_2 units are not responding in the sample of size n. Now, to reduce the effect of non-response, the information is collected by the direct interview on the sub-sampled units of size $m\ (= n_2\,\delta^{-1},\ \delta > 1)$ for $y_i\ (i = 1, 2)$ and following Hansen and Hurwitz (1946), the unbiased estimator

$\bar{y}_{i(HH)}$ [given in section 2 equation (1)] is considered for $\bar{Y}_i$ $(i = 1, 2)$ based on the information of $(n_1 + m)$ units.

When the population means $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_p$ are unknown but estimated by $\bar{x}'_1, \bar{x}'_2, \ldots, \bar{x}'_p$,which is based on larger first phase sample of size n' and we have incomplete information on y_i $(i = 1, 2)$ but complete information on x_j $(j = 1,2, \ldots, p)$ for the sample of size n $(< n')$, then we propose a class of estimators t_p^{**} for P which is given by

$$t_p^{**} = f\left(\prod_{i=1}^{2} \bar{y}_{i(HH)},\ \ (z_1, z_2, \ldots, z_p)'\right) = f(\varphi,\ \boldsymbol{z}'), \tag{28}$$

such that

$$f(P, \boldsymbol{e}') = P, \qquad f_1(P, e') = \left(\frac{\partial}{\partial\varphi} f(\varphi,\ \boldsymbol{z}')\right)_{(P,\ \boldsymbol{e}')} = 1 \tag{29}$$

where $\boldsymbol{z}$ denotes the column vectors $(z_1, z_2, \ldots, z_p)'$ and $z_j = \bar{x}_j / \bar{x}'_j$; $j = 1,2, \ldots, p$.

The function $f(\varphi,\ \boldsymbol{z}')$ satisfies all the necessary regularity conditions similar to those given for the functions $g(\varphi, \boldsymbol{u}')$ and $h(\varphi, \boldsymbol{\omega}')$.

Now, expanding the function $f(\varphi,\ \boldsymbol{z}')$ about the point $(P, \boldsymbol{e}')$ by using Taylor's series up to the second order derivatives, the expressions for bias and mean square error of the estimator t_p^{**} for any sampling design up to the terms of order (n^{-1}) are given by

$$Bias\left(t_p^{**}\right) = Bias\ (\varphi) + E(\varphi - P)(\boldsymbol{z} - \boldsymbol{e})' f_{12}\left(\varphi^*, \boldsymbol{z}^{*'}\right) + \frac{1}{2} E(\boldsymbol{z} - \boldsymbol{e})' h f_{22}(\varphi^*, \boldsymbol{\omega}^{*'})(\boldsymbol{\omega} - \boldsymbol{e}), \tag{30}$$

and

$$MSE\left(t_p^{**}\right) = MSE\ (\varphi) + 2E(\varphi - P)(\boldsymbol{z} - \boldsymbol{e})' f_2(P, \boldsymbol{e}') + E\left(f_2(P, \boldsymbol{e}')\right)'(\boldsymbol{z} - \boldsymbol{e})(\boldsymbol{z} - \boldsymbol{e})'\ f_2(P, \boldsymbol{e}'), \tag{31}$$

where $\boldsymbol{z}^* = \boldsymbol{e} + \emptyset_{\boldsymbol{3}}(\boldsymbol{u} - \boldsymbol{e})$; $0 < \emptyset_{3j} < 1$ and $\emptyset_{\boldsymbol{3}}$ is $(p \times p)$ diagonal matrix having diagonal elements $\emptyset_{3j}$ $(j = 1,2, \ldots, p)$.

Here, $f_1(\varphi, \boldsymbol{z}')$ and $f_2(\varphi, \boldsymbol{z}')$ denote the first partial derivatives of $f(\varphi, \boldsymbol{z}')$ with respect to φ and $\boldsymbol{z}'$ respectively. The second partial derivative of $f(\varphi, \boldsymbol{z}')$ with respect to $\boldsymbol{z}'$ is denoted by $f_{22}(\varphi, \boldsymbol{z}')$ and the first partial derivative of $f_2(\varphi, \boldsymbol{z}')$ with respect to φ is denoted by $f_{12}(\varphi, \boldsymbol{z}')$.

The estimator t_p^{**} will attain the minimum value of the mean square error for

$$f_2(P, \boldsymbol{e}') = -[E(\varphi - P)(\boldsymbol{z} - \boldsymbol{e})' / E(\boldsymbol{z} - \boldsymbol{e})(\boldsymbol{z} - \boldsymbol{e})'] \tag{32}$$

and, the minimum value of mean square error of t_p^{**} is given by

$$MSE\left(t_p^{**}\right)\big|_{min} = MSE\ (\varphi) - \ E(\varphi - P)(\boldsymbol{z} - \boldsymbol{e})'\ [E(\boldsymbol{z} - \boldsymbol{e})(\boldsymbol{z} - \boldsymbol{e})']^{-1} E(\varphi - P)(\boldsymbol{z} - \boldsymbol{e}). \tag{33}$$

To obtain the expressions of bias and the mean square error of t_p^{**} under SRSWOR method of sampling, we assume $\epsilon_j'' = \frac{\bar{x}_j' - \bar{X}_j}{\bar{X}_j}$, such that $E(\epsilon_j'') = 0, |\epsilon_j''| < 1$ and, therefore, we have

$$E(\epsilon_j''^2) = \frac{V(\bar{x}_j')}{\bar{X}_j^2} = \lambda' \frac{S_{x_j}^2}{\bar{X}_j^2}, \quad E(\epsilon_j'', \epsilon_{j'}'') = \frac{Cov(\bar{x}_j', \bar{x}_{j'}')}{\bar{X}_j . \bar{X}_j} = \frac{1}{\bar{X}_j . \bar{X}_{j'}} \left[\lambda' S_{x_j x_{j'}}\right]; \; j \neq j' = 1,2, \ldots, p$$

$$E(\epsilon_{0i}, \epsilon_j'') = \frac{Cov(\bar{y}_{i(HH)}, \bar{x}_j')}{\bar{Y}_i . \bar{X}_j} = \lambda' \frac{S_{y_i x_j}}{\bar{Y}_i . \bar{X}_j}, \qquad E(\epsilon_j', \epsilon_j'') = \frac{Cov(\bar{x}_j, \bar{x}_j')}{\bar{X}_j^2} = \lambda' \frac{S_{x_j}^2}{\bar{X}_j^2},$$

where $\lambda' = \frac{N - n'}{N\, n'}$.

Now, the expressions for bias and the mean square error of t_p^{**} up to the order n^{-1} under SRSWOR method of sampling are given by

$$Bias(t_p^{**}) = Bias\,(\varphi) + \lambda' \left\{P \boldsymbol{b}' f_{12}(\varphi^*, \boldsymbol{z}^{*\prime}) + \frac{1}{2} trace\, \boldsymbol{\mathcal{A}_0}\, f_{22}(\varphi^*, \boldsymbol{z}^{*\prime})\right\} \tag{34}$$

and $$MSE(t_p^{**}) = MSE\,(\varphi) + \lambda' \left\{\left(f_2(P, \boldsymbol{e}')\right)' \boldsymbol{\mathcal{A}_0}\, f_2(P, \boldsymbol{e}') + 2\, P \boldsymbol{b}'\, f_2(P, \boldsymbol{e}')\right\} \tag{35}$$

The conditions for which $MSE(t_p^{**})$ will attain the minimum value are given by

$$f_2(P, \boldsymbol{e}') = P \boldsymbol{\mathcal{A}_0}^{-1} P \tag{36}$$

and the minimum mean square error of t_p^{**} is given by

$$MSE(t_p^{**}) = MSE\,(\varphi) - P^2 \lambda'\, \boldsymbol{b}' \boldsymbol{\mathcal{A}_0}^{-1}\, \boldsymbol{b}. \tag{37}$$

5. Concluding remarks

i) The proposed classes of estimators t_p, t_p^* and t_p^{**} have a wider class of estimators. Following the strategies of Raj (1965), Singh (1967), Abu-Dayyeh *et al.* (2003), Kadilar and Cingi (2005), Perri (2005) and many more, a large number of estimators may be formed, some of them for t_p, t_p^* and t_p^{**} are given in Table 1.

Table 1. Members of classes of estimators t_p, t_p^* and t_p^{**}

$\boldsymbol{t_p}$	$\boldsymbol{t_p^*}$	$\boldsymbol{t_p^{**}}$
$T_{P1} = \varphi \prod_{j=1}^{p} u_j^{\theta_{1j}}$	$T_{P1}^* = \varphi \prod_{j=1}^{p} \omega_j^{\theta_{1j}^*}$	$T_{P1}^{**} = \varphi \prod_{j=1}^{p} z_j^{\theta_{1j}^{**}}$
$T_{P2} = \varphi \prod_{j=1}^{p} u_j^{\theta_{2j}} + \sum_{j=1}^{p} c_j (\bar{X}_j - \bar{x}_{j(HH)})$	$T_{P2}^* = \varphi \prod_{j=1}^{p} \omega_j^{\theta_{2j}^*} + \sum_{j=1}^{p} c_j^* (\bar{X}_j - \bar{x}_{j(HH)})$	$T_{P2}^{**} = \varphi \prod_{j=1}^{p} z_j^{\theta_{2j}^{**}} + \sum_{j=1}^{p} c_j^{**} (\bar{X}_j - \bar{x}_{j(HH)})$
$T_{P3} = \varphi \sum_{j=1}^{p} w_j\, u_j^{\theta_{3j}/w_j}$, $\sum_{j=1}^{p} w_j = 1$	$T_{P3}^* = \varphi \sum_{j=1}^{p} w_j\, \omega_j^{\theta_{3j}^*/w_j}$, $\sum_{j=1}^{p} w_j = 1$	$T_{P3}^{**} = \varphi \sum_{j=1}^{p} w_j\, z_j^{\theta_{3j}^{**}/w_j}$, $\sum_{j=1}^{p} w_j = 1$

Since the estimators (T_{P1}, T_{P2}, T_{P3}), (T_{P1}^*, T_{P2}^*, T_{P3}^*) and (T_{P1}^{**}, T_{P2}^{**}, T_{P3}^{**}) are the members of t_p, t_p^* and t_p^{**} and they satisfy accordingly the conditions (5), (7) and

(29), the values of constants involved in (T_{P1},T_{P2},T_{P3}), (T_{P1}^*,T_{P2}^*,T_{P3}^*) and (T_{P1}^{**},T_{P2}^{**},T_{P3}^{**}) can be calculated by the equations (14), (15) and (32) respectively. In the case when the values of parameters in the optimum value of the constants are not known one may estimate it on the basis of the sample values or may use past data. Srivastava and Jhajj (1983) shown that such values do not affect the mean square error of the estimator up to the terms of order n^{-1} while Reddy (1978) shown that such values are stable over time and region. So the proposed class of two phase sampling estimator is preferred in large scale sample survey.

ii) On comparing the estimators t_p, t_p^* and t_p^{**} with φ in terms of precision, we find

a) $MSE(t_p) < MSE\ (\varphi)$ if
$$MSE\ (\varphi) < \left(g_2(P,e')\right)' \mathcal{A}\ g_2(P,e') + 2\ P\left(\lambda\ \mathscr{b} + \lambda_\delta\ \mathscr{b}_{(2)}\right)' g_2(P,e') < 0$$

b) $MSE(t_p^*) < MSE\ (\varphi)$ if
$$MSE\ (\varphi) < \ \lambda\left\{\left(h_2(P,e')\right)' \mathcal{A}_0\ h_2(P,e') + 2\ P\mathscr{b}'\ h_2(P,e')\right\} < 0$$

c) $MSE(t_p^{**}) < MSE\ (\varphi)$ if
$$MSE\ (\varphi) < \lambda'\left\{\left(f_2(P,\boldsymbol{e}')\right)' \boldsymbol{\mathcal{A}_0}\ f_2(P,\boldsymbol{e}') + 2\ P\boldsymbol{\mathscr{b}}'\ f_2(P,\boldsymbol{e}')\right\} < 0.$$

iii) When there is complete information on the study characters $y_i (i = 1, 2)$ and the auxiliary character and $x_j (j = 1,2, \dots, p)$, i.e. $W_2 = 0$, then we find that the estimators t_p and t_p^* are equally efficient for the class of estimators proposed by Khare (1992) for P using known population means of auxiliary characters. Similarly, the estimator t_p^{**} is also equally efficient to for class of two phase sampling estimators proposed by Khare (1992) for unknown population means of auxiliary characters.

iv) Due to the involvement of various parameters, in the mean square error of t_p and t_p^*, it is very difficult to find the condition for superiority of t_p over t_p^*. However, in the case of one auxiliary character, it has been obtained that relative efficiency of t_p with respect to t_p^* increases by increasing the values of $\frac{\rho_{y_1x_{1(2)}}}{\rho_{y_1x_1}}$ and $\frac{\rho_{y_2x_{1(2)}}}{\rho_{y_2x_1}}$ and decreasing the value of $\frac{\rho_{y_2x_1}}{\rho_{y_1x_1}}$. Hence, one may use the estimators t_p and t_p^* depending upon the values of $\frac{\rho_{y_1x_{j(2)}}}{\rho_{y_1x_j}}$, $\frac{\rho_{y_2x_{j(2)}}}{\rho_{y_2x_j}}$ and $\frac{\rho_{y_2x_j}}{\rho_{y_1x_j}}$ for all $j = 1,2, \dots p$.

6. An empirical study

Source: Police-station – Baria, Tahasil – Champua, District Census Handbook-1981, Orissa, Govt. of India. Total number of villages 109, 25% villages (i.e. 27 villages) from the bottom are considered as the non-responding group of the population. In this data set, study characters and auxiliary characters are as follows:

y_1-Number of occupied residential houses in the village,
y_2-Average number of persons in the house in the village,

x_1-Number of cultivators in the village,
x_2-Area (in hectares) of the village,
x_3-Number of main workers in the village.

The values of the parameters of the population under study are as follows :

$\bar{Y}_1$= 88.3670	$\bar{Y}_2$= 5.5832	$\bar{X}_1$= 100.5505	$\bar{X}_2$= 256.3331	$\bar{X}_3$= 165.2661	
S_{y_1}= 59.3208	S_{y_2}= 0.6024	C_{x_1}= 0.7314	C_{x_2}= 0.6105	C_{x_3}= 0.6828	
$S_{y_{1(2)}}$= 45.2704	$S_{y_{2(2)}}$= 0.5025	$C_{x_{1(2)}}$= 0.5678	$C_{x_{2(2)}}$= 0.4944	$C_{x_{3(2)}}$= 0.5769	
$\rho_{y_1x_1}$= 0.795	$\rho_{y_1x_2}$= 0.854	$\rho_{y_1x_3}$= 0.907	$\rho_{y_2x_1}$= -0.084	$\rho_{y_2x_2}$= -0.117	$\rho_{y_2x_3}$= -0.136
$\rho_{y_1x_{1(2)}}$= 0.658	$\rho_{y_1x_{2(2)}}$= 0.759	$\rho_{y_1x_{3(2)}}$= 0.891	$\rho_{y_2x_{1(2)}}$= 0.092	$\rho_{y_2x_{2(2)}}$= 0.199	$\rho_{y_2x_{3(2)}}$= 0.109
$\rho_{x_1x_2}$= 0.715	$\rho_{x_1x_3}$= 0.841	$\rho_{x_2x_3}$= 0.796	$\rho_{x_1x_{2(2)}}$= 0.541	$\rho_{x_1x_{3(2)}}$= 0.785	$\rho_{x_2x_{3(2)}}$= 0.657
		$\rho_{y_1y_2}$= -0.194	$\rho_{y_1y_{2(2)}}$= 0.023		

To compare the efficiency of the proposed classes of estimators t_p, t_p^* and t_p^{**} with respect to the conventional estimator $\varphi[=\prod_{i=1}^{2}\bar{y}_{i(HH)}]$ through an empirical study based on real data set, their respective members $T_{P1}=\varphi\prod_{j=1}^{p}u_j^{\theta_{1j}}$, $T_{P1}^*=\varphi\prod_{j=1}^{p}\omega_j^{\theta_{1j}^*}$ and $T_{P1}^{**}=\varphi\prod_{j=1}^{p}z_j^{\theta_{1j}^{**}}$ are considered.

The mean square error (MSE) of T_{P1} and T_{P1}^* along with their optimum value of constants ($\boldsymbol{OVC}$) and their percentage relative efficiency (PRE) with respect to φ for different values of sub-sampling fraction ($1/\delta$) are shown in Table 2, while the $MSE(T_{P1}^{**})$ and $PRE(T_{P1}^{**})$ with respect to φ in the case of fixed sample sizes, i.e. $n'=70$ and $n=40$ for different values of sub-sampling fraction ($1/\delta$) are given in Table 3.

7. Discussion and conclusion

From Table 2, it has been observed that the estimators T_{P1} and T_{P1}^* are more efficient than φ [i.e. $\hat{P}$] for the different values of the sub-sampling fraction. We also observe that the mean square error of T_{P1} and T_{P1}^*decreases while the relative efficiency of T_{P1} and $T_{P1}^*$with respect to φ increases as the number of auxiliary characters and sub-sampling fraction increase. From the Table 2, it has also been observed that the estimator T_{P1} is more efficient than T_{P1}^* and the efficiency is increasing with the increase in the number of auxiliary characters and sub-sampling fraction.

From Table 3, we observe that the estimator T_{P1}^{**} is more efficient than φ for the different values of the sub-sampling fraction ($1/\delta$). The relative efficiency of T_{P1}^{**} with respect to φ increases while $MSE(T_{P1}^{**})$ decreases as the number of auxiliary characters and sub-sampling fractions increase. Hence, we conclude that the efficiency of the estimators T_{P1}, T_{P1}^* and T_{P1}^{**} with respect to φ can be increased by increasing the number of the auxiliary characters as well by increasing the values of the sub-sampling fractions.

Table 2. Percentage relative efficiency [PRE (·)] with respect to φ at different values of δ

Estimators	Auxiliary character(s)	$N = 109, n = 40$		
		$1/\delta$		
		$1/4$	$1/3$	$1/2$
φ	–	100.00 (2905.7121)*	100.00 (2494.6591)	100.00 (2083.6062)
T_{P1}	x_1	220.05 (1320.4901)	229.52 (1086.8873)	244.43 (852.4396)
	OVC	θ_{11} = –0.6702	θ_{11} = –0.6802	θ_{11} = –0.6941
	x_1, x_2	411.70 (705.7874)	428.12 (582.7021)	453.38 (459.5749)
	OVC	θ_{11} = –0.3343, θ_{12} = –0.6326	θ_{11} = –0.3369, θ_{12} = –0.6311	θ_{11} = –0.3408, θ_{12} = –0.6286
	x_1, x_2, x_3	691.16 (420.4097)	701.38 (355.6792)	717.60 (290.3568)
	OVC	θ_{11} = –0.015, θ_{12} = –0.382, θ_{13} = –0.575	θ_{11} = –0.028, θ_{12} = –0.385, θ_{13} = –0.565	θ_{11} = –0.048, θ_{12} = –0.388, θ_{13} = –0.549
T^*_{P1}	x_1	157.09 (1849.7092)	173.40 (1438.6562)	202.76 (1027.6033)
	OVC	θ^*_{11} = 0.714	θ^*_{11} = 0.714	θ^*_{11} = 0.714
	x_1, x_2	185.13 (1569.5338)	215.34 (1158.4808)	278.77 (747.4279)
	OVC	θ^*_{11} = –0.348, θ^*_{12} = –0.347	θ^*_{11} = –0.348, θ^*_{12} = –0.347	θ^*_{11} = –0.348, θ^*_{12} = –0.347
	x_1, x_2, x_3	199.42 (1457.1192)	238.48 (1046.0662)	328.12 (635.0133)
	OVC	θ^*_{11} = –0.079, θ^*_{12} = –0.391, θ^*_{13} = –0.523	θ^*_{11} = –0.079, θ^*_{12} = –0.391, θ^*_{13} = –0.523	θ^*_{11} = –0.079, θ^*_{12} = –0.391, θ^*_{13} = –0.523

*****Mean square error of the estimators (·) is shown in the parenthesis.

Table 3. Percentage relative efficiency [PRE (T^{**}_{P1})] with respect to φ at different values of δ for fixed n' and n

Estimators	Auxiliary character(s)	$n' = 70, \ n = 40$		
		$1/\delta$		
		$1/4$	$1/3$	$1/2$
φ	–	100.00 (2905.7121)*	100.00 (2494.6591)	100.00 (2083.6062)
T^{**}_{P1}	x_1	132.74 (2188.9752)	140.31 (1777.9222)	152.44 (1366.8693)
	OVC	θ^{**}_{11} = –0.717	θ^{**}_{11} = –0.717	θ^{**}_{11} = –0.717
	x_1, x_2	144.89 (2005.4099)	156.47 (1594.3569)	176.08 (1183.3040)
	OVC	θ^{**}_{11} = –0.346, θ^{**}_{12} = –0.622	θ^{**}_{11} = –0.346, θ^{**}_{12} = –0.622	θ^{**}_{11} = –0.346, θ^{**}_{12} = –0.622
	x_1, x_2, x_3	150.58 (1929.7012)	164.27 (1518.6482)	188.12 (1107.5953)
	OVC	θ^{**}_{11} = –0.067, θ^{**}_{12} = –0.382, θ^{**}_{13} = –0.538	θ^{**}_{11} = –0.067, θ^{**}_{12} = –0.382, θ^{**}_{13} = –0.538	θ^{**}_{11} = –0.067, θ^{**}_{12} = –0.382, θ^{**}_{13} = –0.538

*Mean square error of the estimators is shown in the parenthesis.

Acknowledgements

Authors are grateful to the referee and the editor for their invaluable suggestions, which helped in further improvisation of the paper.

REFERENCES

ABU-DAYYEH, W. A., AHMED, M. S., AHMED, R. A., MUTTLAK, H. A., (2003). Some estimators of finite population mean using auxiliary information, App. Math. Comp., 139, pp. 287–298.

HANSEN, M. H., HURWITZ, W. N., (1946). The problem of non-response in sample surveys, Jour. Amer. Stat. Assoc., 41, pp. 517–529.

KADILAR, C., CINGI, H., (2005). A new estimator using two auxiliary variables, App. Math. Comp., 162, pp. 901–908.

KHARE, B. B., (1987). On modified class of estimators of ratio and product of two population means using auxiliary character, Proc. Math. Soc., B.H.U., 3, pp. 131–137.

KHARE, B. B., (1990). A generalized class of estimators for a combination of products and ratio of some population means using multi-auxiliary characters, Jour. Stat. Res., 24(1-2), pp. 1–8.

KHARE, B. B., (1991). On generalized class of estimators for ratio of two population means using multi-auxiliary characters, Ali. Jour. Stat., 11, pp. 81–90.

KHARE, B. B., (1992). On class of estimators for product of two population means using multi-auxiliary characters with known and unknown means, Ind. Jour. Appl. Stat., 1, pp. 56–67.

KHARE, B. B., SINHA, R. R., (2002 a). On the class of two phase sampling estimators for the product of two population means using the auxiliary character in presence of non-response, Proceedings of the 5th International Symposium on Optimization and Statistics, pp. 221–232.

KHARE, B. B., SINHA, R. R., (2002 b). Estimation of the ratio of two population means using auxiliary character with unknown population mean in presence of non-response, Prog. Math., 36, pp. 337–348.

KHARE, B. B., SINHA, R. R., (2004 a). Estimation of finite population ratio using two phase sampling scheme in the presence of non-response, Ali. Jour. Stat., 24, pp. 43–56.

KHARE, B. B., SINHA, R. R., (2004 b). On the general class of two phase sampling estimators for the product of two population means using the auxiliary character in the presence of non-response, Ind. Jour. Appl. Stat., 8, pp. 1–14.

KHARE, B. B., SINHA, R. R., (2007). Estimation of the ratio of the two population means using multi-auxiliary characters in the presence of non-response, Proceedings of Statistical Technique in Life Testing, Reliability, Sampling Theory and Quality Control, pp. 205–213.

KHARE, B. B., SINHA, R. R., (2012 a). Improved classes of estimators for ratio of two means with double sampling the non-respondents, Statistika, 49(3), pp. 75–83.

KHARE, B. B., SINHA, R. R., (2012 b). Combined class of estimators for ratio and product of two population means in presence of non-response, Inter. Jour. Stat. Eco., 8, pp. 86–95.

PERRI, P. F., (2005). Combining two auxiliary variables in ratio-cum-product type estimators, Proceedings of Italian Statistical Society, Intermediate Meeting on Statistics and Environment, Messina, 21-23 Sept. 2005, pp. 193–196.

RAJ, D., (1965). On a method of using multi-auxiliary information in sample surveys, Jour. Amer. Stat. Assoc., 60, pp. 154–165.

RAO, P. S. R. S., (1986). Ratio estimation with subsampling the nonrespondents, Survey Methodology, 12(2), pp. 217–230.

RAO, P. S. R. S., (1990). Regression estimators with subsampling of nonrespondents, In-Data Quality Control, Theory and Pragmatics, (Eds.) Gunar E. Liepins and V.R.R. Uppuluri, Marcel Dekker, New York, pp. 191–208.

RAY, S. K., SINGH, R. K., (1985). Some estimators for the ratio and product of population parameters, Jour. Ind. Soc. Ag. Stat., 37, pp. 1–10.

REDDY, V. N., (1978). A study of use of prior knowledge on certain population parameters in estimation, Sankhaya, C, 40, pp. 29–37.

SHAH, S. M., SHAH, D. N., (1978) Ratio cum product estimator for estimating ratio (product) of two population parameters, Sankhya, C, 40, pp. 156–166.

SINGH, H. P., KUMAR, S., (2008). A general family of estimators of finite Population ratio, product and mean using two phase sampling scheme in the presence of non-response, Jour. Stat. Theory and Practice, 2(4), pp. 677–692.

SINGH, H. P., CHANDRA, P., JOARDER, A. H., SINGH, S., (2007). Family of estimators of mean, ratio and product of a finite population using random non-response, Test, 16(3), pp. 565–597.

SINGH, M. P., (1965). On the estimation of ratio and product of population parameters, Sankhya, C, 27, pp. 321–328.

SINGH, M. P., (1967). Ratio cum product method of estimation, Metrika, 12, pp. 34–43.

SINGH, M. P., (1969). Comparison of some ratio cum product estimators, Sankhya, B, 31, pp. 375–378.

SINGH, R. K., (1982 a). Generalized estimators for the estimation of ratio and product of population parameters, Jour. Stat. Res., 16, pp. 15–23.

SINGH, R. K., (1982 b). On estimating ratio and product of population parameters, Cal. Stat. Asso. Bull., 20, pp. 39–49.

SINGH, R. K., (1982 c) Generalized double sampling estimators for the ratio and product of population parameters, Jour. Ind. Stat. Asso., 20, pp. 39–49.

SRIVASTAVA, S. K., JHAJJ, H. S., (1983). A class of estimators of the population mean using multi-auxiliary information, Cal. Stat. Asso. Bull., 32, pp. 47–56.

SRIVASTAVA, S. RANI, KHARE, B. B., SRIVASTAVA, S. R., (1988). On generalised chain estimator for ratio and product of two population means using auxiliary characters, Assam Stat. Rev., 1, pp. 21–29.