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THE GLUEVAR RISK MEASURE AND INVESTOR'S ATTITUDES TO RISK– AN APPLICATION TO THE NON-FERROUS METALS MARKET

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ABSTRACT

Investing in the economic world, characterized by a high level of uncertainty and volatility, entails a higher level of risk related to investment. One of the most commonly used risk measure is Value-at-Risk. However, despite the ease of calculation and interpretation, this measure suffers from a significant drawback – it is not subadditive. This property is the key issue in terms of portfolio diversification. Another risk measure, which meets this assumption, has been proposed – Conditional Value-at-Risk, defined as a conditional loss beyond Value-at-Risk. However, the choice of a risk measure is an individual decision of an investor and it is directly related to his attitudes to risk.
In this paper the new risk measure is proposed – the GlueVaR risk measure, which can be defined as a linear combination of VaR and GlueVaR. It allows for calculating the level of investment loss depending on investment's attitudes to risk. Moreover, GlueVaR meets the subadditivity property, therefore it may be used in portfolio risk assessment. The application of the GlueVaR risk measure is presented for the non-ferrous metals market.

Key words: risk, metal market, subadditivity, VaR, GlueVaR

1. Introduction

In the economic and financial world any disturbances observed in the market as well as additional (non-market) factors affect significantly the level of risk taken. Given the market risk, its level often derives from the investor's behaviour and the way how he assesses the reality around him. The reality is usually different from the assumptions of statistical models, where the most common assumption is the normality of empirical distribution of returns. In the area of financial time series, it is possible to mention certain specific characteristics like significant level of autocorrelation, leptokurtosis, clustering, heavy tails in empirical distributions, etc. These features do not allow for using models based

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on the normality assumption, therefore scientists have to seek for new theoretical solutions to cope with this problem.

The main goal of the paper is the application of the new family of risk measures, called GlueVaR risk measures, to investment risk assessment. The specific type of risk is considered – extreme risk (catastrophic risk), which is related to events with low probability of occurrence, but if they do take place, they can produce large losses (Jajuga 2009). This type of risk is often defined as Low Frequency, High Severity (LFHS), but the precise definition may be represented as in Table 1.

Table 1. Location of risks

Loss	Low probability	High probability
Small	-	regular risk
Large	extreme risk	-

This definition explains that extreme risk is related to its negative perception, where the result of investment generates losses. Theoretical methods used for modelling and examining extreme risk include two popular approaches. The first one is based on the analysis of the distribution of maxima described by the Generalized Extreme Value Theory, and the second one is based on the peaks over threshold (Generalized Pareto Distribution). Extreme risk analysed in this article should be understood more generally. Such risk is considered as related to the event whose probability of occurrence is significantly different from the expected value of the empirical distribution (such models covering these kind of phenomena are within the family of heavy-tailed distributions).

2. Properties of the risk measure

At the very beginning it is necessary to define the measure of risk. Let $\mathbb{X}$ be the set of all random variables defined for a given probability space $(\Omega, \mathcal{A}, P)$. A risk measure ρ is a mapping from $\mathbb{X}$ to $\mathbb{R}$:

$$\mathbb{X} \to \rho(X) \in \mathbb{R}$$

Therefore, $\rho(X)$ is defined as a real value for each $X \in \mathbb{X}$. If the risk measure is defined, certain properties of this measure have to be shown. In 1999 Artzner *et al*. (Artzner *et al*., 1999) presented some axioms describing appropriate risk measure:

- positive homogeneity: $\rho(\lambda X) = \lambda \rho(X)$
- subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
- monotonicity: $X \leq Y \Rightarrow \rho(X) \leq \rho(Y)$
- translation invariance: $\rho\left(X + \alpha R_{free}\right) = \rho(X) - \alpha$

These axioms define a coherent risk measure. The assumptions of positive homogeneity and subadditivity are often replaced by the assumption of convexity:

$$\rho[\lambda X + (1-\lambda)Y] \leq \lambda\rho(X) + (1-\lambda)\rho(Y) \text{for}, \ 0 \leq \lambda \leq 1.$$

Taking into account the investor's point of view, all these axioms are of great importance, but the assumption of subadditivity deserves particular attention. Subadditivity means that the risk of portfolio is equal or lower than the sum of its individual risks. Considering the definition of subadditivity one may link it with diversification, which means that the cumulated risks of individual portfolios cannot be greater than the total risk of the investments. Therefore, the good risk measure should hold these four axioms together.

One of the most popular tools for calculating risk are Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). The advantage of CVaR compared to VaR is that the first one holds the subadditivity assumption and measures the average level of loss in the most adverse cases whereas VaR shows only the minimum loss. The value of CVaR is usually higher than the value of VaR and the selection of risk measure depends on the investor's attitude towards risk.

3. GlueVaR risk measure

The selection of adequate measure of risk and, consequently, the level of risk, is based on underlying investor's attitude to risk. Belles-Sempera*et al*. (Belles-Sempera *et al*., 2014) introduced a new family of risk measures based on Value-at-Risk and Conditional Value-at-Risk namely the GlueVaR risk measure. For a fixed confidence level, the family of the GlueVaR risk measures contains risk measures which lies between the values of VaR and CVaR. Thus, they reflect a particular investor's attitude towards risk. The family of the GlueVaR risk measures is expressed in terms of distortion function and Choquet integral[2]. The distortion function of the GlueVaR risk measure is defined by four-parameter function of the form:

$$\eta_{\gamma_2,\gamma_1}^{m_1,m_2} = \begin{cases} \dfrac{m_1}{1-\gamma_2}u & if \quad 0 \leq u < 1-\gamma_2 \\ m_1 + \dfrac{m_2 - m_1}{\gamma_2 - \gamma_1}[u-(1-\gamma_1)] & if \quad 1-\gamma_2 \leq u < 1-\gamma_1 \\ 1 & if \quad 1-\gamma_1 \leq u \leq 1 \end{cases}$$

where γ_1, γ_2 define confidence levels such that $\gamma_1, \gamma_2 \in [0,1]$ and $\gamma_1 \leq \gamma_2$. Two additional parameters m_1 and m_2 are defined as hits of distortion function such that $m_1 \in [0,1]$and $m_2 \in [m_1, 0]$.

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[2] For more details see Yaari (1987), Choquet (1954), Denneberg (1994).

The GlueVaR risk measure can be expressed in terms of the Choquet integral using the formula:

$$GlueVaR_{\gamma_2,\gamma_1}^{m_1,m_2}(X) = \int X\, d\mu = \int X\, d(\eta_{\gamma_2,\gamma_1}^{m_1,m_2} \circ P)$$

An interesting feature of the GlueVaR risk measure is that it can be expressed as a linear combination of standard risk measures: VaR at the level γ_1, CVaR at the level γ_1 and CVaR at the level γ_2 under the assumption that $0 < \gamma_1 \leq \gamma_2 < 1$):

$$GlueVaR_{\gamma_1,\gamma_2}^{m_1,m_2}(X) = w_1 CVaR_{\gamma_2} + w_2 CVaR_{\gamma_1} + w_3 VaR_{\gamma_1}$$

where weights w_1, w_2 and w_3 are calculated as below:

$$\begin{cases} w_1 = m_1 - \dfrac{(m_2 - m_1)(1 - \gamma_2)}{\gamma_2 - \gamma_1} \\ w_2 = \dfrac{m_2 - m_1}{\gamma_2 - \gamma_1}(1 - \gamma_1) \\ w_3 = 1 - w_1 - w_2 = 1 - m_2 \end{cases}$$

As discussed by Belles-Sempera *et al*., the pairs (m_1, m_2) representing hits of a distortion function of GlueVaR and (w_1, w_2) representing weights given to CVaR at the levels γ_2 and γ_1 respectively are linearly related to each other. The relationship can be expressed in terms of the theory of matrices. Therefore, the relation is as follow:

$$\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \mathrm{H} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \text{ and } \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \mathrm{H}^{-1} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

where matrices H and H^{-1} are of the form:

$$\mathrm{H} = \begin{bmatrix} 1 & \frac{1-\gamma_2}{1-\gamma_1} \\ 1 & 1 \end{bmatrix} \text{ and } \mathrm{H}^{-1} = \begin{bmatrix} \frac{1-\gamma_1}{\gamma_2-\gamma_1} & \frac{\gamma_2-1}{\gamma_2-\gamma_1} \\ \frac{\gamma_1-1}{\gamma_2-\gamma_1} & \frac{1-\gamma_1}{\gamma_2-\gamma_1} \end{bmatrix}$$

For a given parameters w_1 and w_2 we can show special cases of the GlueVaR risk measures:

- if $w_1 = 0$ and $w_2 = 0$ then the GlueVaR risk measures reduce to Value-at-Risk at the level γ_1;
- if $w_1 = 0$ and $w_2 = 1$ then the GlueVaR risk measures reduce to Conditional Value-at-Risk at the level γ_1;
- if $w_1 = 1$ and $w_2 = 0$ then the GlueVaR risk measures reduce to Conditional Value-at-Risk at the level γ_2.

The linear combination of the GlueVaR risk measure allows for defining a particular investor in terms on his attitude towards risk (Belles-Sampera *et al.* 2015). If an investor selects weights $(w_1, w_2) = (1,0)$ then he represents highly conservative attitude towards risk. For the pair $(w_1, w_2) = (0,1)$ he can be defined as conservative. And finally, if he selects weights $(w_1, w_2) = (0,0)$ he is less conservative towards risk. Hence, for given confidence levels γ_1 and γ_2, and for certain levels of weights w_1 and w_2 reflecting the investor's attitude towards risk, the appropriate risk measure within the new family of the GlueVaR risk measures can be selected.

An interesting and attractive feature of the GlueVaR risk measure is that there exist explicit formulas for the most popular probability distributions describing returns. For a normally distributed random variable X, any GlueVaR risk measure can be calculated as:

$$GlueVaR_{\gamma_2,\gamma_1}^{m_1,m_2}(X) = \mu + \sigma q_{\gamma_1}[1 - m_2] + \sigma \frac{m_2 - m_1}{\gamma_2 - \gamma_1}\left[\phi(q_{\gamma_1}) - \phi(q_{\gamma_2})\right] + \sigma \frac{m_1}{1 - \gamma_2}\phi(q_{\gamma_2})$$

where $X \sim N(\mu, \sigma)$, q_{γ_1}, q_{γ_2} represent γ_1 −quantile and γ_2 −quantile of standard normal distribution respectively, and $\phi(\cdot)$ represents the density of standard normal distribution.

If a random variable X is described by t −Student distribution, the expression for the GlueVaR risk measure is of the form:

$$GlueVaR_{\gamma_2,\gamma_1}^{m_1,m_2}(X) = \mu + \sigma[\left(\frac{m_1}{1 - \gamma_2} - \frac{m_2 - m_1}{\gamma_2 - \gamma_1}\right) f(t_{\gamma_2}) \left(\frac{k + t_{\gamma_2}^2}{k - 1}\right) + \frac{m_2 - m_1}{\gamma_2 - \gamma_1} f(t_{\gamma_1}) \left(\frac{k + t_{\gamma_1}^2}{k - 1}\right) + (1 - m_2) t_{\gamma_1}]$$

where $t_{\gamma_1}, t_{\gamma_2}$ represent γ_1 −quantile and γ_2 −quantile of t −Student distribution respectively, k represents degrees of freedom and $f(\cdot)$ represents the density function of t −Student distribution.

4. Empirical analysis on the non-ferrous metals market

The GlueVaR risk measure is applied to assess the risk of investments on the non-ferrous metals market. Due to financial and economic crises observed in the first decade of the 21st century, investors have been forced to search other possibilities to invest capital, which would generate positive returns (Krężołek 2012). The analysis is based on a daily log-returns of spot closing prices of certain non-ferrous metals quoted on the London Metal Exchange from January 2008 to June 2015. The set of assets includes ALUMINIUM, COPPER, LEAD, NICKEL, TIN and ZINC. The quantile-based risk measures such as VaR, CVaR and GlueVaR have been calculated for quantile 0.952 and 0.996, using empirical and theoretical distributions: normal, $t-$Student and $\alpha-$stable. All parameters for theoretical distributions have been calculated using Maximum Likelihood Method. Figures 1-2 present the levels of prices and log-returns for COPPER and ZINC.

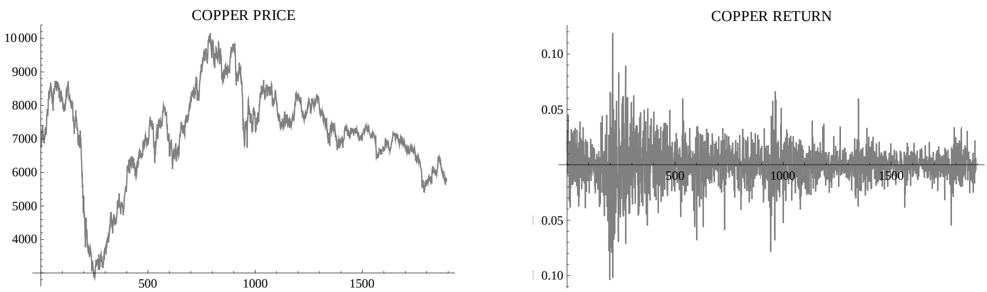

Figure 1. Time series of prices (left) and log returns (right) –COPPER

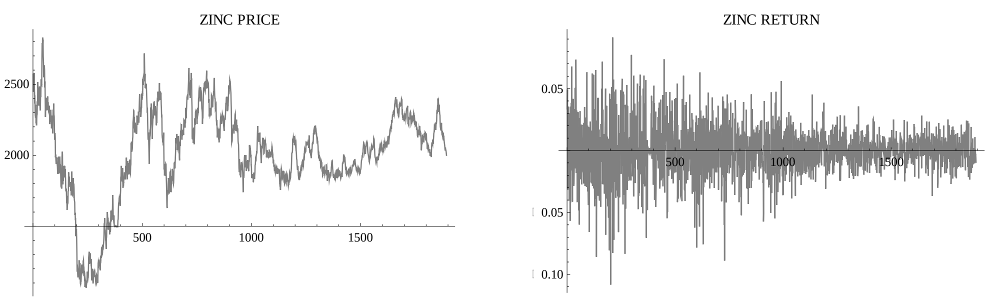

Figure 2. Time series of prices (left) and log returns (right) –ZINC

Figures 1-2 show significant disturbances in price levels, which affect the volatility in log-returns. If log-returns are considered, we can find some specific characteristics of time series, which are very typical for financial assets: clustering of variance, high volatility, long memory effect, etc. In Table 2 certain descriptive statistics of log-returns are presented.

Table 1. Descriptive statistics of log-returns – all metals

Metal/ Statistics	MEAN	STANDARD DEVIATION	KURTOSIS	SKEWNESS	MIN	MAX
ALUMINI UM	-0.00019	0.01492	1.51218	-0.14286	-0.07437	0.05913
COPPER	-0.00008	0.01877	3.80788	-0.11059	-0.10400	0.11880
LEAD	-0.00020	0.02333	3.25932	-0.15558	-0.12850	0.12675
NICKEL	-0.00042	0.02388	3.31503	0.03299	-0.13605	0.13060
TIN	-0.00009	0.01999	4.94503	-0.09549	-0.11435	0.14253
ZINC	-0.00009	0.02047	2.26455	-0.12307	-0.10832	0.09135

The results shown in Table 1 indicate that investments in all analysed metals generate losses. The lowest values of standard deviation are for ALUMINIUM and COPPER. All metals, except NICKEL, are negatively skewed. Moreover, all analysed assets are leptokurtic. This may lead one to assume that empirical distributions are not normal. Goodness-of-fit tests (Anderson-Darling and Cramer-von Misses) have confirmed this hypothesis of non-normality[3]. As an alternative, the t −Student and α −stable distributions have been fitted to the data. The results of estimated parameters are shown in Tables 2-3.

Table 2. Estimated parameters of t −Student distribution*

Metal/ Parameters	$\hat{\mu}$	$\hat{\sigma}$	$\hat{k}$**
ALUMINIUM	-0.00017	0.01250	6.60991
COPPER	0.00007	0.01260	3.28272
LEAD	0.00010	0.01657	3.75685
NICKEL	-0.00058	0.01758	4.18096
TIN	0.00067	0.01253	2.88158
ZINC	-0.00005	0.01497	3.95568

*Maximum Likelihood estimates
**Degrees of freedom

Table 3. Estimated parameters of α −stable distribution*

Metal/ Parameters	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	$\hat{\sigma}$
ALUMINIUM	1.84137	-0.00641	-0.00009	0.00957
COPPER	1.62745	-0.03151	-0.00006	0.01023
LEAD	1.66834	-0.07549	-0.00030	0.01315
NICKEL	1.71689	0.05195	-0.00036	0.01387
TIN	1.56374	-0.21853	-0.00063	0.01027
ZINC	1.68205	0.02343	0.00002	0.01187

*Maximum Likelihood estimates

[3] Due to the length of the paper, some results which are not directly related to the topic are omitted.

The parameters of α −stable distribution allow for indicating additional characteristics of empirical distributions which are not exhibited if normal distribution is considered. The parameter α describes the thickness of tails in empirical distribution, and $\alpha \in (0,2]$. If $alpha < 2$ then variance of the distribution is infinite. The lower values of $alpha$, the thicker tails of empirical distributions. The heaviest tails are for TIN and COPPER, which means that the probability of occurrence of extreme returns is higher than for other metals. If $\alpha = 2$, then the variance is infinite. If $\alpha < 1$ even the mean is infinite. Remaining parameters describe asymmetry ($\beta \in [-1,1]$), location ($\mu \in \mathbb{R}$) and scale of the distribution ($\sigma > 0$).

The main goal of this analysis is to assess the risk using quantile-based risk measures. Assuming two confidence levels $\gamma_1 = 0.952$ and $\gamma_2 = 0.996$ and the set of weights $w = \{w_1, w_2, w_3\}$, three risk measures VaR_α, $CVaR_\alpha$ and $CVaR_\beta$, have been calculated. The confidence level $\gamma_1 = 0.952$ denotes that the extreme event appears twelve times per year4, and $\gamma_1 = 0.996$ denotes that the extreme event appears one time per year. Taking into account the set of weights, seven scenarios have been discussed: $S = \{s_1, s_2, \dots, s_7\}$. The results for ALUMINIUM and ZINC are shown in tables 4-5.

Table 4. Estimated GlueVaR risk measure for ALUMINIUM

	Scenarios						
	s_1	s_2	s_3	s_4	s_5	s_6	s_7
w_1	100.00%	0.00%	0.00%	33.33%	33.33%	66.67%	50.00%
w_2	0.00%	100.00%	0.00%	33.33%	66.67%	33.33%	50.00%
w_3	100.00%	0.00%	0.00%	33.33%	0.00%	0.00%	0.00%
	Distributions						
Empirical distribution	0.04987	0.03253	0.02469	0.03570	0.03831	0.04409	0.04120
Normal distribution	0.03982	0.02972	**0.02480**	0.03145	0.03309	0.03645	0.03477
t-Student distribution	**0.05924**	**0.03249**	0.02397	**0.03857**	**0.04141**	**0.05032**	**0.04586**
α −stable distribution	0.08113	0.03458	0.02317	0.04629	0.05010	0.06561	0.05786

[4] One year is understood as 250 days of trading.

Table 5. Estimated GlueVaR risk measure for ZINC

Scenarios							
	s_1	s_2	s_3	s_4	s_5	s_6	s_7
w_1	100.00%	0.00%	0.00%	33.33%	33.33%	66.67%	50.00%
w_2	0.00%	100.00%	0.00%	33.33%	66.67%	33.33%	50.00%
w_3	100.00%	0.00%	0.00%	33.33%	0.00%	0.00%	0.00%
Distributions							
Empirical distribution	0.07289	0.04775	0.03551	0.05205	0.05613	0.06451	0.06032
Normal distribution	**0.05831**	0.04292	**0.03487**	0.04537	0.04805	**0.05318**	0.05061
t-Student distribution	0.09685	**0.04628**	0.03124	0.05812	0.06314	0.07999	0.07156
α −stable distribution	0.09452	0.04502	0.03145	**0.05699**	**0.06152**	0.07802	**0.06977**

The values in bold in Tables 4-5 represent the estimates of theoretical risk measures closest to empirical ones. As we can find, the closest values are mainly for the heavy-tailed distributions. This finding covers all analysed metals. The results obtained for scenarios s_1, s_2, and s_3 represent the GlueVaR risk measure equal to $CVaR_{\gamma_2}$, $CVaR_{\gamma_1}$and VaR_{γ_1}, respectively. In scenario s_4 we give equal weights to all components of GlueVaR. Scenario s_5 gives higher weight to $CVaR_{\gamma_1}$ and lower to $CVaR_{\gamma_2}$. On the contrary, scenario s_6 - higher weight to $CVaR_{\gamma_2}$ and lower to $CVaR_1$. And finally, scenario s_7 gives equal weights to $CVaR_{\gamma_1}$ and $CVaR_{\gamma_2}$. The location of scenarios in a two-dimensional space of weights is presented in Figure 3.

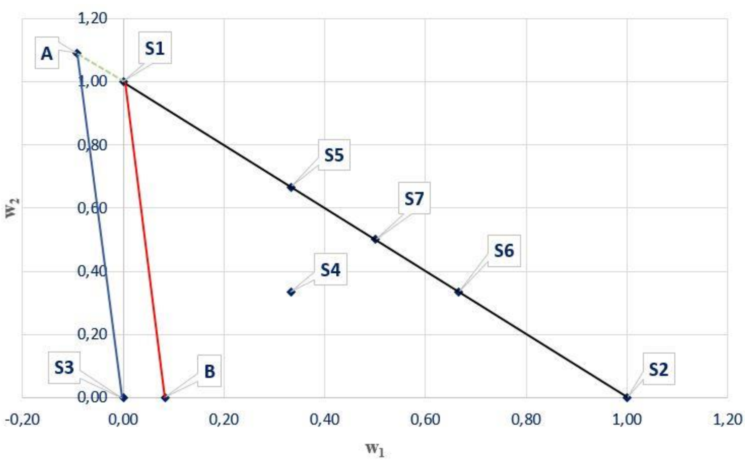

Figure 3. Location of scenarios within the area of feasible weights for GlueVaR risk measures

As discussed in the theoretical part of this paper, the GlueVaR risk measure is associated with confidence levels and weights given to VaR and CVaR. The confidence levels reflect the probability of the occurrence of some extreme events, and weights reflect how much these events are important for a particular investor. To hold the assumption of subadditivity for the GlueVaR risk measure, the weight corresponding to non-subadditive risk component of GlueVaR (i.e. VaR) should meet the relation that$w_3 = 0$. Belles-Sampera *et al.* showed that GlueVaR is subadditive if both weights w_1 and $w_2$belong to the area delimited by the triangle $S1BS2$, especially if they lie on the line segment in a coordinate system described by points: $A = (w_1, w_2) = (\frac{\beta-1}{\beta-\alpha}, \frac{1-\alpha}{\beta-\alpha})$ and $S2 = (w_1, w_2) = (1,0)$ for fixed values of γ_1 and γ_2 $(0 < \gamma_1 \leq \gamma_2 < 1)$. Moreover, the position of a particular point on this line represents the investor's attitude towards risk. The nearer to the point A, the less conservative attitude towards risk. For example, if scenarios s_5 and s_6 are of interest, the values of the GlueVaR risk measure are presented in figure 4.

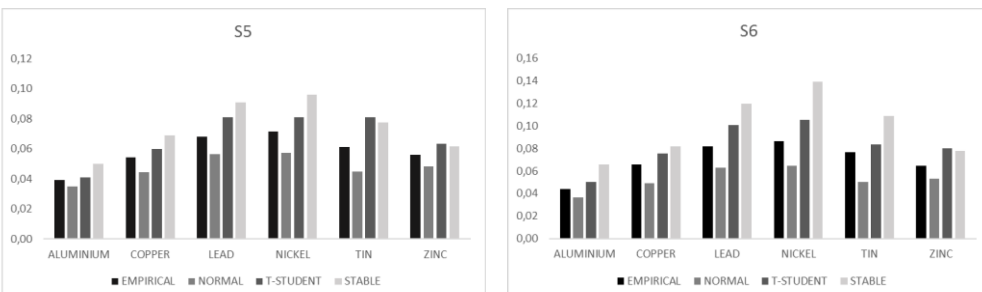

Figure 4. GlueVaR risk measure for scenario s_5 and s_6

The analysis of risk measure provided an interesting conclusion in terms of the use of theoretical distributions. Taking into account fixed confidence levels, despite the fact that the weights are given for each risk measure, the relationship between risk measures and theoretical distributions are presented in Table 6.

Table 6. Relationship between risk measures and theoretical distributions

Theoretical distribution/Risk measure	VaR_{γ_1}	$CVaR_{\gamma_1}$	$CVaR_{\gamma_2}$	$GlueVaR_{\gamma_2,\gamma_1}$
Normal distribution	overestimated	underestimated	underestimated	underestimated
t −Student distribution	underestimated	underestimated	overestimated	overestimated
α −stable distribution	underestimated	underestimated	overestimated	overestimated

The information contained in Table 6 indicates that the normal distribution usually overestimates the value of VaR at the level γ_1 and underestimates remaining risk measures at the levels γ_1 and γ_2. On the other hand, if fat-tailed distributions are considered, the values of VaR and CVaR at the level γ_1 are usually underestimated, while the remaining risk measures at the levels γ_1 and γ_2 are overestimated.

5. Conclusions

In this paper the new family of risk measures, called GlueVaR, has been applied to risk measurement on the non-ferrous metals metal market. This area of investment is not very popular within researchers, although it is a very attractive alternative to classical investments areas (i.e. stocks, exchange rates, etc.). As presented in this paper, some tools for risk assessment used on financial markets can also be used effectively on alternative markets. The methodology of the GlueVaR risk measure is directly related to popular quantile-based risk measures: Value-at-Risk and Conditional Value-at-Risk. These two risk measures determine the value of loss of extreme events. The use of VaR as a risk measure has been impaired due to the failure to meet the assumption of subadditivity. As mentioned before, risk measures such as CVaR and GlueVaR do not suffer such disadvantage. An important feature of the family of the GlueVaR risk measures is that it can be defined as a linear combination of standard risk measures VaR and CVaR for a given confidence levels and for given weights. Taking into account the investor's point of view, the confidence level corresponds to the probability of occurrence of some catastrophic event whereas the weights indicate how such an event is important for the investor. Therefore, a particular investor is able to decide consciously about the acceptable level of risk.

The analysis conducted in this paper is based on both empirical and theoretical distributions (normal, $t-$Student and $\alpha-$stable). The selection of distribution was based on the characteristics of log-returns of the analysed prices of metals. The results show that if the probability of unwanted event is not very low, then the corresponding risk measure should be calculated using normal distribution. Otherwise the fat-tailed distributions are more appropriate. In conclusion, one can say that the family of the GlueVaR risk measures is an attractive and effective tool for risk assessment. This feature results from the subadditivity assumption held for the GlueVaR and from the possibility of considering an individual investor's attitude towards risk. Compared to classical measures, the most useful feature of the proposed new risk measures is that for a particular investor it is possible to implicitly define the set of adverse events and determine the importance of such events. This advantage enables taking into account an individual investor's attitudes towards risk.

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