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SELECTED TESTS COMPARING THE ACCURACY OF INFLATION RATE FORECASTS CONSTRUCTED BY DIFFERENT METHODS

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ABSTRACT

The forecasts of macroeconomic variables including the forecasts of inflation rate play an important role in estimating future situation in the economy. Knowledge of effective forecasts allows making optimal business, financial and investment decisions. The forecasts of macroeconomic variables and as a result also inflation rate forecasts can be determined by different methods often giving different results. Therefore, in this paper we apply selected tests to the evaluation of the accuracy of inflation rate forecasts determined by different methods.

Key words: forecast accuracy, parametric tests, Morgan-Granger-Newbold test, Meese-Rogoff test and Diebold-Mariano test.

1. Introduction

The forecasts of macroeconomic variables and therefore also inflation rate forecasts can be determined by different methods often giving different results (Dittmann, 2008). The purpose of the paper is to apply selected statistical tests to the evaluation of the accuracy of inflation rate forecasts constructed by different methods. Of particular and practical importance are tests which do not need to know the model on which the forecasts that allow comparing the accuracy of forecasts constructed by different methods were determined. This group of parametric tests include: Morgan-Granger-Newbold test, Meese-Rogoff test and Diebold-Mariano test. For this group of tests - the model-free tests - we assume that we have the actual values and the set or sets of forecasts of the prediction.

At the beginning we present the tests which assumes the squared-error loss and zero-mean, serially uncorrelated forecast errors in the context of the application of this tests to the evaluation of the accuracy of inflation rate forecasts determined by different methods. Next, we present tests that are asymptotically valid under more general conditions allowing loss functions other than the

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quadratic and covering situations when forecast errors are non-Gaussian, non-zero-mean, serially correlated, and contemporaneously correlated. These tests are applied also to the evaluation of the accuracy of inflation rate forecasts.

2. Preliminary notions

We assume that the available information consists of the following:

- actual values of the inflation rate $\pi_t, t=1,2,...,T$,
- two forecasts: $\hat{\pi}_{1t}, t=1,2,...,T$ and $\hat{\pi}_{2t}, t=1,2,...,T$.

We define the forecast errors as

$$Q_{it} = \hat{\pi}_{it} - \hat{\pi}_t \text{ for } i=1,2,\ t=1,2,...,T \tag{1}$$

Moreover, we assume that the loss associated with the forecast *i* is a function of the actual and forecast values only through the forecast error, and is denoted by:

$$L(\pi_t, \hat{\pi}_{it}) = L(\hat{\pi}_{it} - \pi_t) = L(Q_{it}) \tag{2}$$

The error loss function *L* can take various forms. Typically, we take into consideration the squared-error loss of the form $L(Q_{it}) = Q_{it}^2$ or the absolute error loss of the form $L(Q_{it}) = \left|Q_{it}\right|$.

We also denote the loss difference between the two forecasts by

$$d_t = L(Q_{1t}) - L(Q_{2t}) \text{ for } t=1,2,...,T \tag{3}$$

Since the tests are presented below verified forecast accuracy, now we define the concept of equality accuracy of inflation rate forecasts. We say that the two inflation rate forecasts have equal accuracy if and only if the loss difference has zero expectation for all *t*.

3. Application of Morgan-Granger-Newbold test to compare the accuracy of inflation rate forecasts

We can apply the Morgan-Granger-Newbold test when the inflation forecasts errors are:

- zero mean,
- Gaussian,
- serially uncorrelated,
- contemporaneously uncorrelated.

Furthermore, we assume the squared-error loss. Moreover, this test is applicable only to one-step predictions.

We would like to test the null hypothesis

$H_0 : E(d_t) = 0$ for all $t = 1, 2, ..., T$

versus the alternative hypothesis

$H_1 : E(d_t) = c \neq 0$.

Therefore, the test statistics is (Diebold, Mariano, 1995, Clements (ed.), Hendry (ed.), 2004):

$$MGN = \frac{r}{\sqrt{\frac{1 - r^2}{T - 1}}} \tag{4}$$

where:

$$r = \frac{x^T \cdot z}{\sqrt{(x^T \cdot x) \cdot (z^T \cdot z)}},$$

x is the $T \times 1$ matrix with *t*-th element x_t,

z is the $T \times 1$ matrix with *t*-th element z_t,

$x_t = Q_{1t} + Q_{2t}$, $z_t = Q_{1t} - Q_{2t}$.

The *MGN* statistics has a t-distribution with $T - 1$ degrees of freedom.

4. Use of Meese-Rogoff test to compare the accuracy of inflation rate forecasts

The Meese-Rogoff test is the test of equal forecast accuracy when the forecast errors are serially and contemporaneously correlated, have zero mean and are Gaussian. In this test we assume also the squared-error loss.

We would like to test the null hypothesis: $H_0 : E(d_t) = 0$ for all $t = 1, 2, ..., T$ versus the alternative hypothesis $H_1 : E(d_t) = c \neq 0$.

Verifying the null hypothesis of equal accuracy of inflation rate forecasts we use also the series: $x_t = Q_{1t} + Q_{2t}$, $z_t = Q_{1t} - Q_{2t}$ for $t = 1, 2, ..., T$.

The statistics for Meese-Rogoff test is then (Rossi, 2005)

$$MR = \frac{\hat{\gamma}_{xz}(0)}{\sqrt{\frac{\hat{\Omega}}{T}}} \tag{5}$$

where:

$$\hat{\gamma}_{xz}(0) = \frac{x^T \cdot z}{T},$$

$$\hat{\Omega} = \sum_{k=-m(T)}^{m(T)} \left(1 - \frac{|k|}{T}\right) \cdot [\hat{\gamma}_{xx}(k) \cdot \hat{\gamma}_{zz}(k) + \hat{\gamma}_{xz}(k) \cdot \hat{\gamma}_{zx}(k)]$$

$\hat{\gamma}_{xz}(k)$, $\hat{\gamma}_{zx}(k)$ - cross-autocovariances,

$\hat{\gamma}_{xx}(k)$, $\hat{\gamma}_{zz}(k)$ - own-autocovariances,

$\hat{\gamma}_{xz}(k) = \text{cov}(x_t, z_{t-k})$

$\hat{\gamma}_{zx}(k) = \text{cov}(z_t, x_{t-k})$

$\hat{\gamma}_{xx}(k) = \text{cov}(x_t, x_{t-k})$

$\hat{\gamma}_{zz}(k) = \text{cov}(z_t, z_{t-k})$

$m(T)$ - the truncation lag that increases with sample size *T*.

Given the maintained assumptions, the following result holds under the hypothesis of equal forecast accuracy $\sqrt{T} \cdot \hat{\gamma}_{xz}(0) \to N(0, \Omega)$ in distribution.

5. Application of Diebold-Mariano test to compare the accuracy of inflation rate forecasts

Diebold and Mariano (1995) consider model-free tests of inflation rate forecast accuracy that are directly applicable to non-quadratic loss functions, multi-period inflation rate forecasts, and inflation rate forecast errors that are non-Gaussian, non-zero-mean, serially correlated, and contemporaneously correlated.

We use this test when sample sizes are large.

The Diebold-Mariano test verify the null hypothesis $H_0 : E(d_t) = 0$ for all $t = 1, 2, ..., T$ versus the alternative hypothesis $H_1 : E(d_t) \neq 0$.

Assuming covariance stationarity of the process d_t, we have the following Diebold-Mariano statistics when the sample size is large (Diebold, Mariano, 1995):

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \cdot \hat{f}_d(0)}{T}}} \tag{6}$$

where:

$$\bar{d} = \frac{1}{T} \sum_{t=1}^{T} [L(Q_{1t}) - L(Q_{2t})]$$

$\hat{f}_d(0)$ is a consistent estimate of $f_d(0)$,

$$\hat{f}_d(0) = \frac{1}{2\pi} \sum_{k=-m(T)}^{m(T)} w\left(\frac{k}{m(T)}\right) \cdot \hat{\gamma}_d(k)$$

$$\hat{\gamma}_d(k) = \frac{1}{T} \sum_{t=|k|+1}^{T} (d_t - \bar{d}) \cdot (d_{t-|k|} - \bar{d})$$

$m(T)$ - the bandwidth or lag truncation that increases with *T*,

$w(\cdot)$ - the weighting scheme or kernel.

One weighting scheme, called the truncated rectangular kernel and used in Diebold and Mariano (1995), is the indicator function that takes the value of unity when the argument has an absolute value less than one, thus $w(x) = 1(|x| < 1)$.

The statistics $\sqrt{T} \cdot (\bar{d} - c) \rightarrow N(0, 2\pi \cdot f_d(0))$ in distribution, where: $f_d(\cdot)$ is the spectral density of d_t for $t = 1, 2, ..., T$, $f_d(\lambda) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_d(k) \cdot e^{-ik\lambda}$ for $-\pi \le \lambda \le \pi$, $\gamma_d(k)$ is the autocovariance of d_t at displacement *k*.

The null hypothesis is rejected in favour of the alternative hypothesis when *DM*, in absolute value, exceeds the critical value of a standard unit Gaussian distribution.

Harvey, Leybourne and Newbold (1997) propose a small-sample modification of Diebold-Mariano test.

When we assume that the inflation rate forecast accuracy is measured in terms of mean-squared prediction error, and optimal *h*-step ahead inflation rate predictions are likely to have forecast errors that are $MA(h-1)$ moving average process of order $h-1$, we have autocovariances $\gamma(k) = 0$ for $k \ge h$ and $\hat{\gamma}_d(k) = \frac{1}{T} \sum_{t=k+1}^{T} (d_t - \bar{d}) \cdot (d_{t-k} - \bar{d})$ for $0 \le k \le h$.

Then, the test statistics that is the modification of *DM* test statistics, is the following (Harvey, Leybourne, Newbold, 1997):

$$DM^* = \frac{DM}{\sqrt{\dfrac{T + 1 - 2h + \dfrac{h(h-1)}{T}}{T}}} \tag{7}$$

To make a decision of rejection or acceptance of the null hypothesis the empirical value with critical value from the t-distribution with $(T-1)$ degrees of freedom should be compared.

6. Empirical analysis

Below we present the results of testing inflation rate forecast accuracy for monthly inflation rate determined on the basis of the autoregressive model and the traditional VAR monetary policy model, and also for the quarterly inflation rate that come from the reports "Inflation projection of the NBP based on the NECMOD model".

6.1. Comparison of the accuracy of inflation rate forecasts for the forecasts obtained from the autoregressive model and from the traditional VAR monetary policy model

When testing the equality of inflation rate forecasts accuracy we take into account one-step forecast monthly inflation rates determined on the basis of the first-order autoregression model and on the basis of the traditional VAR monetary policy model, which contains three variables: inflation rate, industrial production growth rate and reference rate. We assume the significance level equals 0,01. The data concerning the monthly forecast of inflation rates, the real values of inflation rate and the forecast errors are presented in the table below.

Table 1. Forecasts and real values of monthly inflation rate and forecast errors

Time	Inflation forecasts determined on the basis of the first-order autoregression model	Inflation forecasts determined on the basis of the traditional VAR monetary policy model	Real values of inflation rate	Q_{1t}	Q_{2t}
April 2011	4.59	4.55	4.5	0.09	0.05
May 2011	4.71	4.33	5	-0.29	-0.67
June 2011	4.76	4.83	4.2	0.56	0.63
July 2011	4.78	4.9	4.1	0.68	0.8
August 2011	4.79	4.54	4.3	0.49	0.24

Source: Own calculations.

Since the inflation forecasts errors do not have zero mean ($\overline{Q}_{1t}$ = 0.306, $\overline{Q}_{2t}$ = 0.21), then to test the equality of forecasts accuracy we use the modification of Diebold-Mariano test for a small sample proposed by Harvey, Leybourne and Newbold.

The null hypothesis and alternative hypothesis are the following:

$H_0 : E(d_t) = 0$ for all $t = 1, 2, \ldots, 5$

$H_1 : E(d_t) \neq 0$

The test statistics is given by (7)

$$DM^* = \frac{DM}{\sqrt{\frac{T + 1 - 2h + \frac{h(h-1)}{T}}{T}}}$$

Assuming $m(T) = 3$ we obtain DM = -1.28 and DM^*_{emp} = -1.43. Because the critical value read from the table of t-distribution with 4 degrees of freedom is equal to $DM_\alpha = t_{0,01;4} = 4,604$, then for this significance level we have $\left| DM^*_{emp} \right| < DM_\alpha$, thus there is no evidence to reject the null hypothesis of equal forecast accuracy of monthly inflation rates determined on the basis of the first-order autoregression model and on the basis of the traditional VAR monetary policy model.

6.2. Comparison of the accuracy of inflation rate forecasts and "Inflation projection of the NBP based on the NECMOD model"

When comparing the accuracy of inflation rate forecasts we now take into account forecasts of quarterly inflation rates provided in the report "Inflation projection of the NBP based on the NECMOD model". We assume the significance level equals 0.01.

The obtained data concerning the quarterly forecasts of inflation rate, the real values of inflation rate, the forecast errors and the loss difference are presented in the tables below.

Table 2. Forecasts and real values of quarterly inflation rate

Year	Quarter	Inflation forecasts from given report	Inflation forecasts from the next report	Real values of inflation rate
2008	I	4.2	4.3	4.3
	II	4.6	4.7	4.7
	III	3.8	3.8	3.8
2009	I	3.4	3.3	3.3
	II	3.3	3.7	3.7
	III	3.6	3.6	3.5
	IV	3.0	3.3	3.3

Table 2. Forecasts and real values of quarterly inflation rate (cont.)

Year	Quarter	Inflation forecasts from given report	Inflation forecasts from the next report	Real values of inflation rate
2010	I	2.6	3.0	3.0
	II	2.4	2.3	2.3
	III	2.1	2.2	2.2
	IV	2.9	2.9	2.9
2011	I	3.5	3.8	3.8
	II	4.3	4.6	4.6
	III	4.1	4.1	4.1
	IV	4.6	4.6	4.6
2012	I	4.3	4.1	4.1
	II	3.9	4.0	4.0
	III	3.9	3.8	3.9
	IV	3.1	2.9	2.9
2013	I	1.7	1.3	1.3
	II	1.4	0.6	0.5

Source: Report "Inflation projection of the NBP based on the NECMOD model".

Table 3. The forecast errors and the loss difference

Year	Quarter	Q_{1t}	Q_{2t}	Loss difference d_t
2008	I	0.0	-0.1	-0.01
	II	0.0	-0.1	-0.01
	III	0.0	0.0	0.00
2009	I	0.0	0.1	-0.01
	II	0.0	-0.4	-0.16
	III	0.1	0.1	0.00
	IV	0.0	-0.3	-0.09
2010	I	0.0	-0.4	-0.16
	II	0.0	0.1	-0.01
	III	0.0	-0.1	-0.01
	IV	0.0	0.0	0.00
2011	I	0.0	-0.3	-0.09
	II	0.0	-0.3	-0.09
	III	0.0	0.0	0.00
	IV	0.0	0.0	0.00
2012	I	0.0	0.2	-0.04
	II	0.0	-0.1	-0.01
	III	-0.1	0.0	0.01
	IV	0.0	0.2	-0.04
2013	I	0.0	0.4	-0.16
	II	0.1	0.9	-0.80

Source: Own calculations.

In this case to compare the forecasts accuracy we use the modification of Diebold-Mariano test for a small sample.

The null hypothesis and the alternative hypothesis are as follow:

$H_0 : E(d_t) = 0$ for all $t = 1, 2, ..., 21$

$H_1 : E(d_t) \neq 0$

The test statistics is given by (7)

Assuming $m(T) = 5$ we obtain $DM = 2{,}22$ and $DM^*_{emp} = 2{,}39$. Because the critical value read from the table of t-distribution with 20 degrees of freedom is equal to $DM_\alpha = t_{0,01;\,20} = 2{,}845$, then for the significance level which equals 0.01 we have $\left|DM^*_{emp}\right| < DM_\alpha$. Subsequently, there is no evidence to reject the null hypothesis of equal forecast accuracy of quarterly forecasts of inflation rate determined on the basis of NECMOD model. Therefore, all determined forecasts have equal accuracy. The differences in values result from the change in the assumptions about the projections in the individual reports.

7. Conclusion

It follows from the analyses that the most frequently used test for the comparison of the accuracy of inflation rate forecasts (the forecasts constructed by different methods) is the modification of Diebold-Mariano test for a small sample proposed by Harvey, Leybourne and Newbold. It can be concluded that there is no evidence to reject the null hypothesis of equal forecast accuracy of monthly inflation rates determined on the basis of the first-order autoregression model and on the basis of the traditional VAR monetary policy model. We also conclude that the quarterly forecasts of inflation rate determined on the basis of NECMOD model and presented in two subsequent reports have equal accuracy. The differences in values result from the change in the assumptions about the projections in the individual reports.

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