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# IMPROVED SEPARATE RATIO AND PRODUCT EXPONENTIAL TYPE ESTIMATORS IN THE CASE OF POST-STRATIFICATION

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## ABSTRACT

This paper addressed the problem of estimation of finite population mean in the case of post-stratification. Improved separate ratio and product exponential type estimators in the case of post-stratification are suggested. The biases and mean squared errors of the suggested estimators are obtained up to the first degree of approximation. Theoretical and empirical studies have been done to demonstrate better efficiencies of the suggested estimators than other considered estimators.

**Key words:** finite population mean, post-stratification, bias, mean squared error.

## 1. Introduction

The problem of post-stratification was first discussed by Hansen et al. (1953). Ige and Tripathi (1989) studied the properties of classical ratio and product estimators of population mean in the case of post-stratification. Chouhan (2012) studied the Bahl and Tuteja (1991) estimators in the case of post-stratification. Many researchers including Kish (1965), Fuller (1966), Raj (1972), Holt and Smith (1979), Agrawal and Pandey (1993), Lone and Tailor (2014), Jatwa (2014), Lone and Tailor (2015), Tailor et al. (2015) contributed significantly to this area of research.

Bahl and Tuteja (1991) envisaged a ratio and a product type exponential estimator of population mean in simple random sampling. Following Srivenkataramana (1980) and Bondyopadhyayh (1980), Lone and Tailor (2014, 2015) proposed dual to separate ratio and product type exponential estimators in the case of post-stratification.

Let us consider a finite population $U = (U_1, U_2, ..., U_N)$. A sample of size *n* is drawn from population $U$ using simple random sampling without replacement.

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After selecting the sample, it is observed which units belong to $h^{th}$ stratum. Let $n_h$ be the size of the sample falling in $h^{th}$ stratum such that $\sum_{h=1}^{L} n_h = n$. Here, it is assumed that $n$ is so large that the possibility of $n_h$ being zero is very small.

Let $y_{hi}$ be the observation on $i^{th}$ unit that fall in $h^{th}$ stratum for study variate $y$ and $x_{hi}$ be the observation on $i^{th}$ unit that fall in $h^{th}$ stratum for auxiliary variate $x$, then

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$ : $h^{th}$ stratum mean of the study variate $y$,

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$ : $h^{th}$ stratum mean of the auxiliary variate $x$,

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \sum_{h=1}^{L} W_h \bar{Y}_h$ : Population mean of the study variate $y$ and

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h \bar{X}_h$ : Population mean of the auxiliary variate $x$.

In the case of post-stratification, the usual unbiased estimator of population mean $\bar{Y}$ is defined as

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{1.1}$$

where

$W_h = \frac{N_h}{N}$ is the weight of the $h^{th}$ stratum and $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ is the sample mean of $n_h$ sample units that fall in the $h^{th}$ stratum.

Using the results from Stephen (1945), the variance of $\bar{y}_{PS}$ to the first degree of approximation is obtained as

$$Var(\bar{y}_{PS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1 - W_h) S_{yh}^2 \tag{1.2}$$

where $S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2$ .

Separate ratio and product type estimators of population mean $\bar{Y}$ in the case of post-stratification are defined as

$$\hat{\bar{Y}}_{RPS} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h}\right) \tag{1.3}$$

and

$$\hat{\bar{Y}}_{PPS} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{z}_h}{\bar{Z}_h}\right). \tag{1.4}$$

Up to the first degree of approximation, biases and mean squared errors of the estimators $\hat{\bar{Y}}_{RPS}$ and $\hat{\bar{Y}}_{PPS}$ are obtained as

$$B\left(\hat{\bar{Y}}_{RPS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} \bar{Y}_h \left(C_{xh}^2 - \rho_{yxh} C_{xh} C_{yh}\right), \tag{1.5}$$

$$MSE\left(\hat{\bar{Y}}_{RPS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_{1h}^2 S_{xh}^2 - 2\sum_{h=1}^{L} W_h R_{1h} S_{yxh}\right], \tag{1.6}$$

$$B\left(\hat{\bar{Y}}_{PPS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} \bar{Y}_h \, C_{yh} \, C_{zh} \, \rho_{yzh} \tag{1.7}$$

and

$$MSE\left(\hat{\bar{Y}}_{PPS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h \, R_{2h}^2 S_{zh}^2 + 2\sum_{h=1}^{L} W_h \, R_{2h} S_{yzh}\right], \tag{1.8}$$

where $R_{1h} = \frac{\bar{Y}_h}{\bar{X}_h}$ and $R_{2h} = \frac{\bar{Y}_h}{\bar{Z}_h}$ .

## 2. Improved separate ratio exponential type estimator

We suggest the improved separate ratio exponential type estimator for population mean $\bar{Y}$ in the case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{(a_h)} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + (a_h - 1)\,\bar{x}_h}\right), \tag{2.1}$$

where $a_h \geq 0$.

To obtain the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{PS}^{(a_h)}$, we write

$$\bar{y}_h = \bar{Y}_h\left(1+e_{0h}\right),\ \ \bar{x}_h = \bar{X}_h\left(1+e_{1h}\right) \text{ such that}$$

$$E(e_{0h}) = E(e_{1h}) = 0,$$

$$E(e_{0h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{yh}^2,$$

$$E(e_{1h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{xh}^2,$$

$$E(e_{0h}e_{1h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{yxh}C_{yh}C_{xh}.$$

Expressing (2.1) in terms of $e_{ih}$'s, we have

$$\hat{\bar{Y}}_{PS}^{(a_h)} = \sum_{h=1}^{L} W_h\bar{Y}_h\left(1+e_{0h}\right)\exp\left[-\frac{e_{1h}}{a_h}\left\{1+\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}^{-1}\right]$$

Now, by expanding the exponential function on the right-hand side, we get

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h\bar{Y}_h\left(1+e_{0h}\right)\left[1-\frac{e_{1h}}{a_h}\left\{1+\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}^{-1} + \frac{1}{2}\frac{e_{1h}^2}{a_h^2}\left\{1+\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}^{-2} - \ldots\right]$$

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h\bar{Y}_h\left(1+e_{0h}\right)\left[1-\frac{e_{1h}}{a_h}\left\{1-\left(\frac{a_h-1}{a_h}\right)e_{1h} + \left(\frac{a_h-1}{a_h}\right)^2 e_{1h}^2\right\} + \frac{1}{2}\frac{e_{1h}^2}{a_h^2}\left\{1-2\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}\right]$$

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h\bar{Y}_h\left(1+e_{0h}\right)\left[1-\frac{e_{1h}}{a_h} + \frac{e_{1h}^2}{a_h^2}\left(a_h-\frac{1}{2}\right)\right]$$

$$\left(\hat{\bar{Y}}_{PS}^{(a_h)} - \bar{Y}\right) = \sum_{h=1}^{L} W_h\bar{Y}_h\left[e_{0h} - \frac{e_{1h}}{a_h} + \left(a_h-\frac{1}{2}\right)\frac{e_{1h}^2}{a_h^2} - \frac{e_{0h}e_{1h}}{a_h}\right] \tag{2.2}$$

Now, taking expectation of both sides of (2.2), the bias of the suggested estimator $\hat{\bar{Y}}_{PS}^{(a_h)}$ to the first degree of approximation is obtained as

$$B(\hat{\bar{Y}}_{PS}^{(a_h)}) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} \frac{1}{a_h^2 \, \bar{X}_h} \left[\left(a_h - \frac{1}{2}\right) R_{1h} \, S_{xh}^2 - a_h S_{yxh}\right] \tag{2.3}$$

Squaring both sides of (2.2) and then taking expectation, we get the mean squared error of the suggested estimator $\hat{\bar{Y}}_{PS}^{(a_h)}$ up to the first degree of approximation as

$$MSE\left(\hat{\bar{Y}}_{PS}^{(a_h)}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{R_{1h}^2}{a_h^2} S_{xh}^2 - 2 \frac{R_{1h}}{a_h} S_{yxh}\right) \tag{2.4}$$

which is minimized for

$$a_h = \frac{R_{1h}}{\beta_h} = a_{ho} \quad (say), \tag{2.5}$$

where $R_{1h} = \frac{\bar{Y}_h}{\bar{X}_h}$ and $\beta_h = \frac{S_{yxh}}{S_{xh}^2}$.

Putting (2.5) in (2.4), we get the minimum mean squared error of the estimator $\hat{\bar{Y}}_{PS}^{(a_h)}$ up to the first degree of approximation as

$$\min . MSE\left(\hat{\bar{Y}}_{PS}^{(a_h)}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h \, S_{yh}^2 \left(1 - \rho_h^2\right), \tag{2.6}$$

where $\rho_h = \frac{S_{yxh}}{S_{yh} \, S_{xh}}$.

Putting (2.5) in (2.1), we get the asymptotic optimum estimator (AOE) in the class of estimators $\hat{\bar{Y}}_{PS}^{(a_h)}$ as

$$\hat{\bar{Y}}_{PS}^{(a_{h0})} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\beta_h \, (\bar{X}_h - \bar{x}_h)}{\beta_h \bar{X}_h + (R_{1h} - \beta_h) \, \bar{x}_h}\right). \tag{2.7}$$

## 3. Estimator based on estimated optimum

It is obvious that the estimator $\hat{\bar{Y}}_{PS}^{(a_{h0})}$ in (2.7) requires prior information of $(R_{1h}, \beta_h)$, which can be obtained easily from previous surveys. If the investigator is unable to guess the value of $(R_{1h}, \beta_h)$, the only alternative he is left with is to replace $(R_{1h}, \beta_h)$ in (2.7) by its consistent estimate $\hat{a}_h = \frac{\hat{R}_{1h}}{\hat{\beta}_h}$, where $\hat{\beta}_h = \frac{s_{yxh}}{s_{xh}^2}$ and $\hat{R}_{1h} = \frac{\bar{y}_h}{\bar{x}_h}$. Hence, the estimator based on estimated optimum is

$$\hat{\bar{\bar{Y}}}_{PS}^{(\hat{a}_{h0})} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left( \frac{\hat{\beta}_h (\bar{X}_h - \bar{x}_h)}{\bar{y}_h + \hat{\beta}_h (\bar{X}_h - \bar{x}_h)} \right) \tag{3.1}$$

Up to the first degree of approximation, the mean squared error of the estimator $\hat{\bar{\bar{Y}}}_{PS}^{(\hat{a}_{h0})}$ is given by

$$MSE\left(\hat{\bar{\bar{Y}}}_{PS}^{(\hat{a}_{h0})}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yh}^2 \left(1 - \rho_h^2\right) \tag{3.2}$$

which is the same as given in (2.6)

## 4. Efficiency comparisons of the suggested improved ratio exponential type estimator $\hat{\bar{\bar{Y}}}_{PS}^{(a_h)}$ with $\hat{\bar{\bar{Y}}}_{PS}$ and $\hat{\bar{\bar{Y}}}_{RPS}$.

From (1.2), (1.6) and (2.4), it is observed that the suggested estimator $\hat{\bar{\bar{Y}}}_{PS}^{(a_h)}$ would be more efficient than

(i) the usual unbiased estimator $\hat{\bar{\bar{Y}}}_{PS}$ if

$$\sum_{h=1}^{L} \frac{R_{1h}}{a_h^2} W_h \left(R_{1h} S_{xh}^2 - 2 a_h S_{yxh}\right) < 0 \ , \tag{4.1}$$

(ii) the usual separate ratio estimator $\hat{\bar{\bar{Y}}}_{RPS}$ if

$$\sum_{h=1}^{L} W_h \left( R_{1h}^2 S_{xh}^2 \left\{ \frac{1}{a_h^2} - 1 \right\} - 2 S_{yxh} R_{1h} \left\{ \frac{1}{a_h} - 1 \right\} \right) < 0 \, . \tag{4.2}$$

## 5. Improved separate product exponential type estimator

Improved separate product exponential type estimator for population mean $\bar{Y}$ in the case of post-stratification is being suggested as

$$\hat{\bar{\bar{Y}}}_{PS}^{(b_h)} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left( \frac{\bar{z}_h - \bar{Z}_h}{\bar{Z}_h + (b_h - 1)\, \bar{z}_h} \right), \tag{5.1}$$

where $b_h \geq 0$.

The estimator $\hat{\bar{Y}}_{PS}^{(b_h)}$ in terms of $e's$ can be written as

$$\left(\hat{\bar{Y}}_{PS}^{(b_h)} - \bar{Y}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \left[ e_{0h} + \frac{e_{2h}}{b_h} + \left(\frac{3}{2} - b_h\right)\frac{e_{2h}^2}{b_h^2} + \frac{e_{0h}e_{2h}}{b_h} \right] \tag{5.2}$$

Using the standard procedure, the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{PS}^{(b_h)}$ are obtained as

$$B(\hat{\bar{Y}}_{PS}^{(b_h)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} \frac{W_h}{\bar{Z}_h}\left(\left(\frac{3}{2} - b_h\right)\frac{R_{2h}^2}{b_h^2} S_{zh}^2 + \frac{1}{b_h} S_{yzh}\right) \tag{5.3}$$

and

$$MSE\left(\hat{\bar{Y}}_{PS}^{(b_h)}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{R_{2h}^2}{b_h^2} S_{zh}^2 + 2\frac{R_{2h}}{b_h} S_{yzh}\right) \tag{5.4}$$

which is minimized for

$$b_h = -\frac{R_{2h}}{\beta_h^*} = b_{ho} \ (say), \tag{5.5}$$

where $\rho_h^* = \dfrac{S_{yzh}}{S_{yh}\ S_{zh}}$ and $\beta_h^* = \dfrac{S_{yzh}}{S_{zh}^2}$.

Putting (5.5) in (5.4), we get the minimum mean squared error of the estimators $\hat{\bar{Y}}_{PS}^{(b_h)}$ to the first degree of approximation given as

$$\min.MSE\left(\hat{\bar{Y}}_{PS}^{(b_h)}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\, S_{yh}^2 \left(1 - \rho_h^{*2}\right) \tag{5.6}$$

Putting (5.5) in (5.1), we get the asymptotic optimum estimator (AOE) in the class of estimators $\hat{\bar{Y}}_{PS}^{(b_h)}$ as

$$\hat{\bar{Y}}_{PS}^{(b_{h0})} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\beta_h^*(\bar{z}_h - \bar{Z}_h)}{\beta_h^* \bar{Z}_h - (R_{2h} + \beta_h^*)\,\bar{z}_h}\right), \tag{5.7}$$

with the same mean square as given in (5.6)

## 6. Estimator based on estimated optimum value of $b_{ho}$

If the value of $(R_{2h}, \beta_h^*)$ is not known in advance, then it is advisable to replace them by its consistent estimate $(\hat{R}_{2h}, \hat{\beta}_h^*)$ computed from the sample values. Hence, the estimator based on estimated optimum is

$$\hat{\bar{Y}}_{PS}^{(\hat{b}_{h0})} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left( \frac{\hat{\beta}_h^* (\bar{Z}_h - \bar{z}_h)}{\bar{y}_h + \hat{\beta}_h^* (\bar{z}_h - \bar{Z}_h)} \right) \quad (6.1)$$

The mean squared error of the estimator $\hat{\bar{Y}}_{PS}^{(\hat{b}_{h0})}$ up to the first degree of approximation is given by

$$MSE\left(\hat{\bar{Y}}_{PS}^{(\hat{a}_{h0})}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yh}^2 \left(1 - \rho_h^{*2}\right) \quad (6.2)$$

which is the same as given in (5.6)

## 7. Efficiency comparisons of the suggested estimator $\hat{\bar{Y}}_{PS}^{(b_h)}$ with $\hat{\bar{Y}}_{PS}$ and $\hat{\bar{Y}}_{PPS}$

From (1.2), (1.8) and (5.4), it is concluded that the suggested estimator $\hat{\bar{Y}}_{PS}^{(b_h)}$ would be more efficient than

(i) the usual unbiased estimator $\hat{\bar{Y}}_{PS}$ if

$$\sum_{h=1}^{L} W_h \frac{R_{2h}}{b_h^2} \left(R_{2h} S_{zh}^2 + 2 b_h S_{yzh}\right) < 0 \quad (7.1)$$

(ii) the usual separate product estimator $\hat{\bar{Y}}_{PPS}$ if

$$\sum_{h=1}^{L} W_h \left( \left\{ \frac{1}{b_h^2} - 1 \right\} R_{2h}^2 S_{zh}^2 + 2 \left\{ \frac{1}{b_h} - 1 \right\} R_{2h} S_{yzh} \right) < 0 . \quad (7.2)$$

## 8. Empirical study

To judge the performance of the suggested estimators we are considering two natural population data sets, the descriptions of populations are given below:

**Population I- [Source: National horticulture Board]**

$y$ : Productivity (MT/Hectare)

$x$ : Production in '000 Tons and

$z$ : Area in '000 Hectare

| Constant | Stratum I | Stratum II |
|---|---|---|
| $N_h$ | 10 | 10 |
| $n_h$ | 4 | 4 |
| $\bar{Y}_h$ | 1.70 | 3.67 |
| $\bar{X}_h$ | 10.41 | 289.14 |
| $\bar{Z}_h$ | 6.32 | 80.67 |
| $S_{yh}$ | 0.50 | 1.41 |
| $S_{xh}$ | 3.53 | 111.61 |
| $S_{zh}$ | 1.19 | 10.82 |
| $S_{yxh}$ | 1.60 | 144.87 |
| $S_{yzh}$ | -0.05 | -7.04 |
| $S_{xzh}$ | 1.38 | -92.02 |

**Population II- [Source: Chouhan (2012)]**

$y$ : Snowy days

$x$ : Rainy days and

$z$ : Total annual sunshine hours

| Constant | Stratum I | Stratum II |
|---|---|---|
| $N_h$ | 10 | 10 |
| $n_h$ | 4 | 4 |
| $\bar{Y}_h$ | 149.7 | 102.6 |
| $\bar{X}_h$ | 142.8 | 91.0 |
| $\bar{Z}_h$ | 1629.9 | 2035.9 |
| $S_{yh}$ | 13.46 | 12.60 |
| $S_{xh}$ | 6.09 | 6.57 |
| $S_{zh}$ | 102.17 | 103.26 |
| $S_{yxh}$ | 18.44 | 23.30 |
| $S_{yzh}$ | -1072.8 | -655.25 |
| $S_{xzh}$ | -239.25 | -240.45 |

**Table 8.1.** Percent Relative Efficiencies of $\hat{\bar{Y}}_{PS}$, $\hat{\bar{Y}}_{RPS}$, $\hat{\bar{Y}}_{PPS}$, $\hat{\bar{Y}}_{PS}^{(a_{h0})}$ and $\hat{\bar{Y}}_{PS}^{(b_{h0})}$ with respect to $\hat{\bar{Y}}_{PS}$

| Estimators | Percent Relative Efficiencies (PRE's) | |
|---|---|---|
| | Population I | Population II |
| $\hat{\bar{Y}}_{PS}$ | 100.00 | 100.00 |
| $\hat{\bar{Y}}_{RPS}$ | 593.50 | 98.72 |
| $\hat{\bar{Y}}_{PPS}$ | 116.84 | 176.97 |
| $\hat{\bar{Y}}_{PS}^{(a_{h0})}$ | **643.41** | **106.82** |
| $\hat{\bar{Y}}_{PS}^{(b_{h0})}$ | **123.44** | **179.35** |

## 9. Conclusion

Section 4 and 7 provides the conditions under which the suggested estimators $\hat{\bar{Y}}_{PS}^{(a_{h0})}$ and $\hat{\bar{Y}}_{PS}^{(b_{h0})}$ have fewer mean squared errors in comparison with usual unbiased estimator and separate ratio and product type estimators in the case of post-stratification. Table 8.1 shows that the suggested estimators $\hat{\bar{Y}}_{PS}^{(a_{h0})}$ and $\hat{\bar{Y}}_{PS}^{(b_{h0})}$ have higher percent relative efficiencies in comparison with usual unbiased estimator $\hat{\bar{Y}}_{PS}$, separate ratio and product type estimators $\hat{\bar{Y}}_{RPS}$ and $\hat{\bar{Y}}_{PPS}$. Thus, the suggested estimators are recommended for use in practice for estimating the population mean when conditions obtained in section 4 and 7 are satisfied.

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