STATISTICS IN TRANSITION new series, Summer 2014
Vol. 15, No. 3, pp. 427–436

THE POSITION OF THE WIG INDEX IN COMPARISON WITH SELECTED MARKET INDICES IN BOOM AND BUST PERIODS

Anna Czapkiewicz[1], **Beata Basiura**[2]

ABSTRACT

The main aim of this work is to discover the differences between the rank of Polish stock market in the boom and the bust cycles. The data of the daily stock market returns for the twenty three major international indices from Europe, America and Asia are used in the research. Two boom and two bust periods are considered. The correlation coefficient obtained from Copula-GARCH model is a similarity measure between the considered indices returns. The cluster analysis carried on for these series in the boom and bust the cycles allows us to find the differences in the market behaviour.

Key words: clustering stock indices, dependence parameter, Copula-GARCH model.

1. Introduction

Finding similarities between world financial markets has been one of the primary intention amongst investigations. Practitioners are interested in identifying these similarities to assess investment risk. Knowledge about market relationships enables us to diversify this risk. To gain insight into the internal relationship between financial time series the cluster analysis has proved useful, producing a set of markets grouped according to a given measure of similarity in their behaviour. However, in the case of the clustering time series we encounter difficulties with the choice of an appropriate measure which could be used as a measure of similarity between the indices returns and takes into account the character of the considered series. It is known that the function of the Pearson correlation coefficient as a measure of similarity between pairs of stock returns (Mantegna, 1999; Bonanno et al., 2001) is not a satisfactory measure of

[1] Faculty of Management, AGH University of Science and Technology, Krakow, Poland, A. Mickiewicza 30 Ave., 30-059. E-mail: gzrembie@cyf-kr.edu.pl.
[2] Faculty of Management, AGH University of Science and Technology, Krakow, Poland, A. Mickiewicza 30 Ave., 30-059. E-mail: bbasiura@zarz.agh.edu.pl.

dependence. Without the multivariate normality assumption, two pairs of markets can have equal linear correlation coefficient while they can still differ in terms of dependence structure.

A useful tool to describe dependence between time series is the application of copula function to model the multivariate distribution (Embrechts et al. 2001, 2003). Copulas are useful to apply because they allow us to separate the dependence properties of the data from their marginal properties and to construct multivariate models with marginal distributions of an arbitrary form. Some of them are appropriate for financial markets. The most popular are *t*-Student and Joe-Clayton copulas. The first one is recommended, for example, by Mashal, Zevi (2002) and Breymann et al. (2003). The AR(1)-GARCH(1,1) model with skewed *t*-Student conditional distribution is quite satisfactory for describing the indices returns behaviour and may be applied as the marginal. The elements of the correlation matrix obtained from *t*-Student copula may be considered as the similarity measure between data. Having the matrix of distances based on these the similarity measure the Ward algorithm (Ward 1968) may be used to cluster the indices into the similar groups. The applicability of such a methodology for grouping global markets was presented in the work by Czapkiewicz, Basiura (Czapkiewicz, Basiura 2010).

This study concerns the determination of the Poland's position in comparison with the selected market indices. Empirical study covers two boom periods and two bust periods from June 2003 to March 2012. The possibility of difference in grouping of the markets in the boom and the bust periods is taken into consideration. It is anticipated that the negative moods in the stock markets strongly influence the other markets than the positive ones. The aim of this empirical work is to search for the differences in the relation strength of Polish market with other markets in the boom and bust periods. Furthermore, the clustering of the twenty three markets is carried on in these periods. These periods are defined according to WIG and WIG 20 indices behaviour.

2. The model

2.1. The distributions of returns

The advantage of using copulas, as mentioned in the introduction, stems from the fact that marginal distributions can be separated from the underlying dependency structure. Many models have been proposed to describe the dynamics of return. In this paper we consider the univariate AR(1)-GARCH(1,1) model. It is defined as follows:

$$y_t = \mu + \alpha y_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sqrt{h_t}\eta_t$$
$$h_t = a_0 + a_1\varepsilon_{t-1}^2 + a_2 h_{t-1}, \quad \eta_t \sim iid(0,1)$$

In the above equation y_t denotes the daily return of stock market index. Scrutiny of daily returns led to the introduction of fat-tailed distributions for this residuals. Fat-tails are not the only problem in the context of conditional distribution, the skewness can also be noticed. That is why the skewed distribution was considered as the conditional distribution in the above process.

2.2. The copula function

The copula function is a multivariate distribution defined on the unit cube $[0,1]^d$, with uniformly distributed margins. The function $C: [0,1]^d \to [0,1]$ is a d-dimensional copula if it satisfies the following properties:

1. For all $u_i \in [0,1]$, $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$.
2. For all $u \in [0,1]^d, C(u_1, \dots, u_d) = 0$, if at least one coordinate $u_i = 0$.
3. C is d-increasing.

The importance of the copula function stems from the fact that it captures the dependence structure of the multivariate distribution. According to Sklar's theorem (Sklar, 1959) a given d-dimensional distribution function F with margins $F_1, \dots, F_d$ can be presented as:

$$F(x) = C\big(F_1(x_1), \dots, F_d(x_d)\big).$$

When $F(x)$ is a multivariate continuous distribution function of a random vector $X \in R^d$ and $F_i(x_i)$ are continuous margins the copula is uniquely determined.

The copula used in the empirical part is the *t*-Student copula:

$$C(u_1, \dots, u_d) = t_{\Sigma,\eta}\left(t_\eta^{-1}(u_1), \dots t_\eta^{-1}(u_d)\right)$$

where t_η is the *t*-Student's cumulative distribution with η degrees of freedom and $t_{\Sigma,\eta}$ is the *t*-Student's cumulative distribution with η degrees of freedom and the correlation matrix Σ. The bivariate case of *t*-Student copula is given by:

$$C(u_i, u_j; \rho_{ij}) = \int_{-\infty}^{t_\eta^{-1}(u_i)} \int_{-\infty}^{t_\eta^{-1}(u_j)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{s^2 - 2\rho_{ij} st + t^2}{\eta(1-\rho_{ij}{}^2)}\right)^{-\frac{\eta+2}{2}} ds\, dt$$

where ρ_{ij} is the correlation ratio.

The Copula-GARCH model may be estimated by maximum likelihood method. The IFM strategy (Shih, Louis, 1995; Joe, Xu, 1996) is used for this purpose in the empirical work. IFM proceeds in the two steps. Firstly, the parameter estimates of margins distribution are obtained, secondly, the estimate of copula dependence parameter[1] is calculated. Under some regularity conditions, Patton (2006) shows that the IFM procedure yields consistent and asymptotically normal estimates.

[1] It is the commonly used name of the parameter ρ_{ij}, because it measures dependence between the marginals (cf. P. K. Trivedi and D. M. Zimmer (2005)).

3. Empirical study

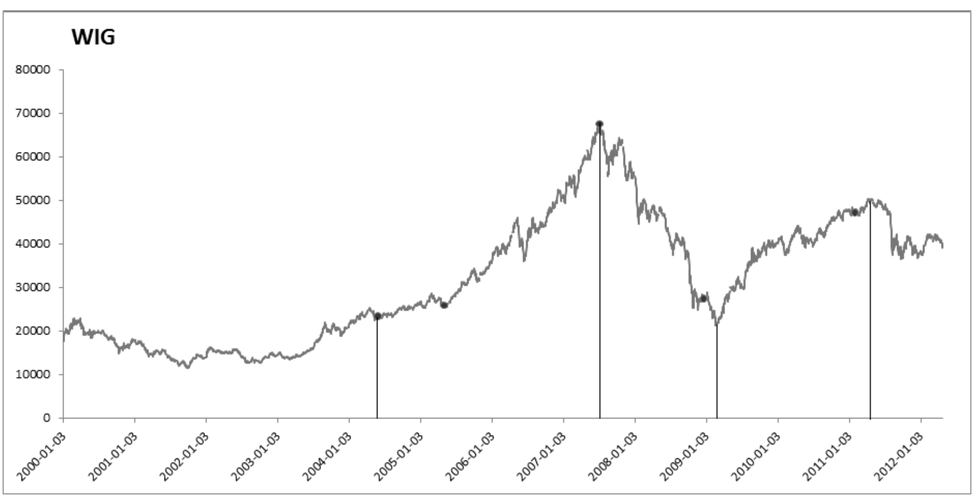
3.1. The data

The study covers the period from June 2003 to March 2012. During this period four sub-periods are extracted. The choice of these sub-periods is related to WIG and WIG 20 trading. The first - from July 2003 to June 2007 is a boom sub-period; the second - from August 2007 to February 2009 - the bust one; the third - a post crisis sub-period, where both indices WIG and WIG20 rise again (from February 2009 to July 2011). The last sub-period (from July 2011 to January 2012) is determined as the bust one. Figure 1 and Figure 2 present the trading WIG index and the trading WIG20 index in the searching sub-periods.

The relationships between some selected stock indices are investigated separately in each sub-period. We will compare the stock markets in the following countries on the basis of the indices given in brackets:

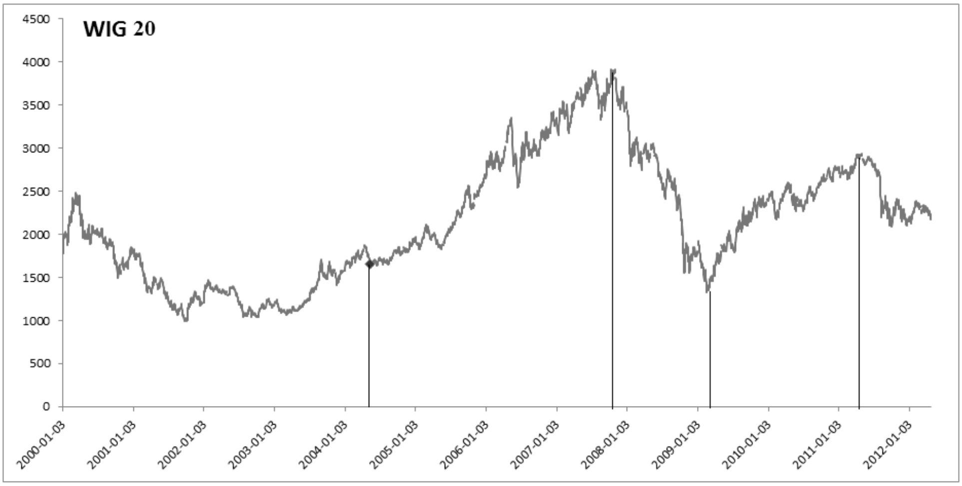
Poland (WIG and WIG20), Austria (ATX), Euronext Brussels (BEL20), Bulgaria (SOFIX), Canada (TSX), China (HSI), Czech Republic (PX), Finland (HEX), France (CAC40), Germany (DAX), Hungary (BUX), Japan (NIKKEI), Norway (OSE), Romania (BET), Russia (RTS), Slovakia (SAX), South Korea (KOSPI), Spain (IBEX), Switzerland (SMI), the Netherlands (AEX), the UK (FTM), the USA (DJIA), Turkey (ISE).

Figure 1. The trading WIG index from January 2002 to March 2012.



Source: gpw.pl, April 2012.

Figure 2. The trading WIG20 index from January 2002 to March 2012.



Source: gpw.pl, April 2012.

The investigation covers returns of market indices from the world. It includes the markets deemed as emerging as well as those already developed of North America, Europe and Asia. In the case of the USA, the DJIA index is taken into consideration. In addition, the BEL20, the benchmark stock market index of Euronext Brussels, is included in the research.

It should be noted that the indices selected for testing represent wide or narrow market. So, to meet these constrains two indices: WIG and WIG20 are chosen as representatives of Polish market.

The daily frequency data are taken into study. Missing data are filled by linear interpolation from the preceding to the following missing quotations. The return of indices is defined as $r_t = \ln(P_t / P_{t-1})$ where P_t is an adjusted index value at period t.

Some tests for conditional heteroskedasticity and autocorrelation are performed. The results of Engle test led us to assume that the choice of the GARCH model is justified while the Ljung-Box test results indicate the possibility of the autocorrelation presence. For all considered cases GARCH effect and the autocorrelation exist. These results are the reason for the introduction of AR(1)-GARCH(1,1) model to describe the indices returns behaviour.

3.2. Estimation of the multivariate model

In a preliminary step of our empirical work, we investigate the structure of the univariate marginal returns. The AR(1)-GARCH(1,1) model with the skewed t-Student's conditional distribution is considered to describe returns modelling.

Thus, the procedure of testing the goodness-of-fit is carried out. For the testing purposes, we follow the procedure described in Diebold et al. (1998). If a marginal distribution is correctly specified, the margins denoting the transformed standardized AR(1) - GARCH(1,1) residuals should be *iid* Uniform (0,1). For the most of the analyzed time series the test results confirm the correctness of the chosen model.

Prior to the main study, a preliminary analysis of relationship between WIG and WIG20 is carried out. The study is conducted using data from the whole sample. The estimated parameter of *t*-Student copula ($\rho = 0.98$) indicates a very strong correlation between these two indexes.

This strong relationship makes very small differences between the parameters defining the dependence between the Polish market and other markets if we consider the WIG20 index instead of the WIG index. So, taking pairs of the Polish index with an index representing the market of another country, the parameters of the bivariate *t*-Student copula are estimated. If an index represents a narrow market the WIG20 index is a representative of the Polish market.

Table 1 presents the correlation coefficients obtained from *t*-Student copula for Polish index with other indices considered in the boom and bust sub-periods. According to intuition, one would expect the dependencies with other markets should be greater during the boom sub-periods than during the bust sub-periods.

Table 1. The correlation coefficients obtained from *t*-Student copula for Polish index with other indices considered in the boom and bust periods

Country	1^{th} boom	1^{th} bust	2^{nd} boom	2^{nd} bust
Austria	0.39	**0.67**	**0.61**	**0.72**
Belgium	0.43	**0.66**	**0.63**	**0.76**
Bulgaria	0.05	0.19	0.26	0.27
Canada	0.27	0.36	0.43	**0.57**
China	0.08	0.15	0.21	0.15
Czech Republic	**0.46**	**0.68**	**0.66**	**0.67**
Finland	0.43	**0.60**	**0.63**	**0.77**
France	**0.44**	**0.67**	**0.68**	**0.77**
Germany	0.40	**0.65**	**0.62**	**0.77**
Hungary	**0.54**	**0.64**	**0.64**	**0.64**
Japan	0.28	0.35	0.27	0.34
Norway	0.43	**0.55**	**0.65**	**0.73**
Romania	0.06	0.41	0.44	**0.49**
Russia	0.42	**0.56**	**0.64**	**0.68**
Slovakia	0.04	-0.05	-0.03	0.04
South Korea	0.33	0.34	0.35	0.43
Spain	0.43	**0.63**	**0.61**	**0.68**
Switzerland	0.40	**0.62**	**0.61**	**0.70**
the Netherlands	**0.46**	**0.66**	**0.68**	**0.76**
the UK	**0.48**	**0.67**	**0.66**	**0.75**
the USA	0.21	0.34	0.48	**0.60**
Turkey	0.36	**0.66**	0.55	**0.68**

The results confirm our prediction. The analysis of the estimated correlation coefficients indicates that the relationship of Polish market with other markets is stronger during the bust sub-periods than during the boom ones. The beginning of the first considered sub-period (as a reminder it is from June 2003 to July 2007) precedes the date of Poland's entry into the European Union. Under this sub-period study the strongest correlation is observed only in the case of the Hungarian index. The relations with markets of the Eurozone are relatively weak. The correlation between Polish and Russian indices is similar to the correlations between Polish index and indices of Western Europe. It is noted that no significant relations of the Polish market with markets of Slovakia, Romania, Bulgaria or China exist.

The stronger dependence between Poland and other markets is observed in the first bust sub-period. The Polish index is strongly correlated with indices of Czech Republic, Austria, Belgium, France, Germany, the Netherlands, the UK and Turkey (coefficients are greater than 0.65).

In the second boom sub-period, quite strong correlation is observed between the Polish index and the indices of the Netherlands, France, Czech Republic, the UK, Norway, Hungary, Russia, Belgium, Finland, Germany, Austria, Spain and Switzerland (correlation coefficients are greater than 0.60).

In the second bust sub-period the strength of relationship of Poland with other markets increases. In this sub-period there are the strongest dependences between the Polish and other studied markets. At that time Romania and Bulgaria are in the European Union, so a stronger correlation with those markets indices is found than in the previous sub-periods. At the same time Poland consolidates its position in the European Union so much stronger relationship of Poland with the Eurozone countries is observed.

It is worth noting that in the second boom sub-period the correlations between Poland and other markets are not significantly lower than under the previous boom sub-period study. So, one might conclude that the strength of relationships between markets increases, regardless of the economic situation.

For all studied periods the weakest correlation is noted between Polish and Slovakian indices. It seems that Slovakian market is not linked with Polish market at all.

A more complete picture of links between the markets is obtained using the clustering method based on these determined parameters. Although the correlation coefficient might be relatively high, the index might belong to another cluster. In the following part of the empirical study the results of grouping of indices in the considered periods are presented. The Ward algorithm is adopted for this purpose with the dissimilarity measure $d_{ij} = 1 - \rho_{ij}$. Figure 3 presents the dendrograms for two boom periods. Figure 3a shows the results of grouping of indices for the first boom period, while Figure 3b presents clustering results for the second boom one. A similar analysis is performed for data from the two bust periods. Figure 4a shows the clustering results for the period of global crisis, while Figure 4b presents the results for the data of the last bust period.

Figure 3. The dendrograms for market indices in the two periods of boom; a) from July 2003 to June 2007; b) from February 2009 to July 2011

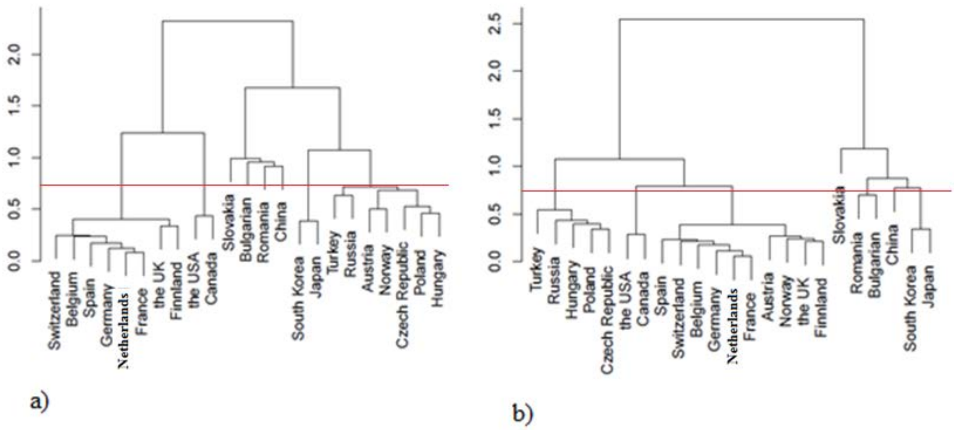
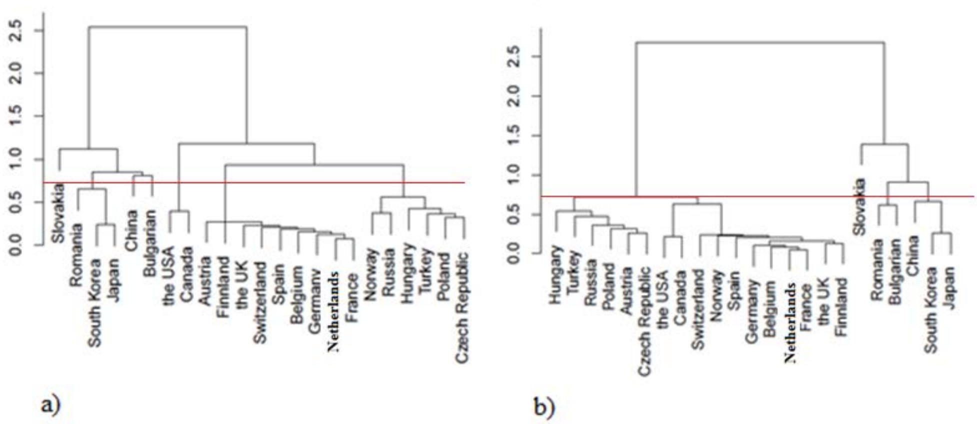


Figure 4. The dendrograms for market indices in the two periods of bust; a) from August 2007 to February 2009; b) from July 2011 to January 2012



For this research it is very important that the group included data obtained with a strong dependence (ρ_{ij} close to one). So, it is assumed that the tree should be cut for $d = 0{,}75$. It can be noticed that clusters are being varied during the considered periods. In the first boom period there are four groups. First group concentrate markets from the Eurozone: Switzerland, Belgium, Spain, Germany, the Netherlands and France. The markets of Finland and the UK stand out against the background of European markets. The USA and Canada markets belong to one group. The Japan market is grouped with South Korea one. A separate group consists of markets from North, Central and Eastern Europe. There are three abstracted subgroups: first - Turkey and Russia, second - Austria and Norway -

and last - Poland, Hungary and Czech Republic. The isolated markets are: Slovakia, Bulgaria, Romania and China. When the dendrogram from the second boom period is analyzed similar grouping can be noticed.

In the first bust sub-period, the indices of Austria, Finland, the UK, Switzerland, Spain, Belgium, Germany, the Netherlands and France are in the one subgroup, whereas indices of the Norway, Russia, Hungary, Turkey, Poland and Czech Republic are in the other. The rest of the analyzed indices form separate clusters. Similar grouping is observed in the second bust period.

The Polish index is in the same subset as Hungarian, Czech Republic, Turkish and Russian indices regardless of the studying sub-periods.

4. Conclusions

The purpose of this paper is to investigate the relationships of Polish market with some European markets and main markets of America and Asia. As a measure of the relationship between the markets the correlation coefficient obtained from the *t*-Student copula is used. Returns are modelled by AR(1)-GARCH(1,1) process. The study is conducted for the four sub-periods: two boom periods and two bust periods. These sub-periods are defined on the basis the WIG and WIG20 indices trading.

The empirical results indicate that the relationship of Polish index with other indices is stronger during the bust sub-periods than during the boom ones. Furthermore, it is noted that the strength of the relationship between Polish market and others increased, regardless of the situation on the stock markets. The relationships between the Polish market and other markets may be affected by many factors, of which by Poland's entry into the European Union.

As the results show the clustering methods yield different groupings depending on the considered sub-periods. The groupings in the boom sub-periods seem to be similar to each other although in the case of the second boom period, the binding to other markets took place on the lower levels (as evidenced by stronger correlation coefficients). In the bust period, relatively large number of markets seemed to be in one class. Polish index occurs in one subset with Hungarian, Czech Republic, Turkish and Russian indices, regardless of the studied sub-periods.

REFERENCES

BASIURA, B., CZAPKIEWICZ, A., (2010). *Clustering Financial Data Using Copula-GARCH Model In an Application for Main Market Stock Returns*, Statistics in Transition (New Series), Poland, Vol. 11, No. 1, pp. 25–45.

BONANNO, G., LILLO, F., MANTEGNA, R. N., (2001). *High-frequency cross-correlation in a set of stocks*, *Quantitative Finance* 1, pp. 96–104.

BREYMANN, W., DIAS ,A., EMBRECHTS, P., (2003). *Dependence Structures for Multivariate High-Frequency Data in Finance*, Quantitative Finance, 3, pp. 1–14.

DIEBOLD, F. X., GUNTHER, T. A., TAY, A. S., (1998). *Evaluating Density Forecasts with Applications to Financial Risk Management*, International Economic Review, 39(4): pp. 863–883.

EMBREECHT, P., MCNEIL, A. J., STRAUMANN, D., (2001). *Correlation and dependency in risk management: properties and pitfalls*, In: M. Dempster, H. Moffant, *Risk Management*, Cambridge University Press, New York, pp. 176–223.

EMBRECHTS, P., LINDSKOG, F., MCNEIL, A., (2003). *Modeling Dependence with Copulas and Applications to Risk Management*, In: Rachev, S.T. (Ed.), *Handbook of Heavy Tailed Distributions in Finance*, Elsevier/North-Holland, Amsterdam.

JOE, H., XU, J. J., (1996). *The estimation method of inference function for margins for multivariate models*, Technical Report, Departments of Statistics, University of British Columbia.

MASHAL, R., ZEEVI, A., (2002). *Beyond Correlation: Extreme co-movements Between Financial Assets*, Mimeo, Columbia Graduate School of Business.

MANTEGNA, R. N., (1999). *Hierarchical structure in financial markets*, *European Physical Journal* B 11, pp. 193–197.

MIRKIN, B., (2005). *Clustering for Data Mining: A Data Recovery Approach*, Boca Raton Fl., Chapman and Hall/CRC.

PATTON, A. J., (2006). Estimation of multivariate models for time series of possibly different lengths, *Journal of Applied Econometrics*, John Wiley & Sons. Ltd., 21(2): pp. 147–173.

ROSENBLATT, M., (1952). Remarks on a Multivariate Transformation, *The Annals of Mathematical statistics*, 23, pp. 470–472.

R Development Core Team, (2004). R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, Austria, ISBN is 3-900051-07-0 URL http://www.Rproject.org.

SHIH, J., LOUIS, T. A., (1995). Inference on the Association Parameter in Copula Models for Bivariate Survival Data, *Biometric*s, 51: pp. 1384-1399.

SKLAR, A., (1959). Fonction de Repartition a n Dimension et Leur Marges, *Publications de L'Institut de Statistiques de L'Universite de Paris*, 8, pp. 229–231.

TRIVEDI, P. K., ZIMMER, D. M., (2005). *Copula Modeling: An Introduction for Practitioners*, Foundations and Trends in Econometrics, Vol. 1, No 1, pp. 1–111, ed. Now, the Essence of Knowledge.

WARD, J. H., (1963). Hierarchical Grouping to Optimize an Objective Function, *Journal of the American Statistical Association*, 58, pp. 236–244.