

LAG LENGTH SPECIFICATION IN ENGLE-GRANGER COINTEGRATION TEST: A MODIFIED KOYCK MEAN LAG APPROACH BASED ON PARTIAL CORRELATION

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ABSTRACT

The Engle-Granger cointegration test is highly sensitive to the choice of lag length and the poor performance of conventional lag selection criteria such as standard information criteria in selecting appropriate optimal lag length for the implementation of the Engle-Granger cointegration test is well-established in the statistical literature. Testing for cointegration within the framework of the residual-based Engle-Granger cointegration methodology is the same as testing for the stationarity of the residual series via the augmented Dickey-Fuller test which is well known to be sensitive to the choice of lag length. Given an array of candidate optimal lag lengths that may be suggested by different standard information criteria, the applied researchers are faced with the problem of deciding the best optimal lag among the candidate optimal lag lengths suggested by different standard information criteria, which are often either underestimated or overestimated. In an attempt to address this well-known major pitfall of standard information criteria, this paper introduces a new lag selection criterion called a modified Koyck mean lag approach based on partial correlation criterion for the selection of optimal lag length for the residual-based Engle-Granger cointegration test. Based on empirical findings, it was observed that in some instances over-specification of lag length can bias the Engle-Granger cointegration test towards the rejection of a true cointegration relationship and non-rejection of a spurious cointegration relationship. Using real-life data, we present an empirical illustration which demonstrates that our proposed criterion outperformed the standard information criteria in selecting appropriate optimal truncation lag for the implementation of the Engle-Granger cointegration test using both augmented Dickey-Fuller and generalized least squares Dickey-Fuller tests.

Key words: modified Koyck mean lag, partial correlation criterion, Engle-Granger cointegration test, optimal truncation lag, information criteria, augmented Dickey-Fuller test, generalized least square Dickey-Fuller test.

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1. Introduction

The residual-based Engle-Granger cointegration methodology is arguably the most widely used bivariate cointegration test in empirical analysis. One of the major specification decisions that poses a big challenge to analysts and applied researchers is selection of appropriate lag length for the implementation of unit root test for the estimated residuals from cointegrating regressions. A number of previous studies have demonstrated a strong influence of lag selection on the outcome of the Engle-Granger cointegration test. Gutierrez et al. (2009) show that misspecification of appropriate lag length may greatly affect the cointegration results such that under-specification of lag length could invalidate the cointegration test and over-specification of lag length could result in a loss of power. Hall (1991) pointed out that the choice of lag structure in the error correction model (ECM) is a vital specification decision because too few lags may lead to serial correlation problem, whereas too many lags specified in the ECM will consume more degree of freedoms leading to small sample problem. Li et al. (2009) also corroborated Hall (1991) position by arguing that appropriate specification of lag length is one of the most important specification decisions concerning implementation of the error correction process. Johansen (1991) proposed the use of appropriate information criterion or a sequence of likelihood ratio tests for the determination of lag length.

This paper is primarily concerned with appropriate specification of lag length for the cointegration test as well as the error correction process (ECP) within the context of the Engle-Granger cointegration methodology. Standard information criteria such as Akaike Information Criterion (AIC), Akaike Final Prediction Error (FPE), the Bayesian Information Criterion (BIC) and Hannan-Quinn Information Criterion (HQIC) that are commonly employed for the choice of optimal lag structure have been shown to exhibit a strong tendency to either over-specify or under-specify the lag length. Nishi (1988) and Lutkepohl (1993) showed that both Akaike Information Criterion (AIC) and Final Prediction Error (FPE) are not consistent estimators of the truncation lag order but the Bayesian Information Criterion (BIC) is strongly consistent. Bewley and Yang (1998) evaluated the performance of standard information criteria such as AIC and BIC in selecting appropriate lag structure for the cointegration test and showed that these conventional lag selection criteria appear to have problem of underestimation and overestimation of the lag structure. Clarke and Mirza (2006) argue that both AIC and FPE cannot be recommended as lag selection procedures since both criteria are well known to have a positive probability of overestimating the true lag order.

In general, the major drawback of the commonly used standard information criteria lies with problem of underestimation and overestimation of lag length which are regarded as undesirable in cointegration analysis as demonstrated in Cheung and Lai (1993) and Gonzalo (1994). Given this demonstrated weaknesses of the standard information criteria, we therefore propose an alternative lag

selection criterion called a modified Koyck mean lag approach based on partial correlation criterion (MK-PCC) for the purpose of lag specification in the residual-based Engle-Granger cointegration test proposed by Engle and Granger (1987).

The remaining part of this paper is organized as follows. Section 2 discusses specification of augmented Dickey-Fuller (ADF) and generalized least squares Dickey-Fuller (DF-GLS) tests for the implementation of the residual-based Engle-Granger cointegration test. Section 3 introduces lag specification procedure based on the modified Koyck mean lag approach using partial correlation criterion. Section 4 presents preliminary data description and unit root tests. Section 5 discusses the Engle-Granger cointegration tests, residual analysis and estimation of error correction models. Finally, section 6 concludes.

2. Specification of Engle-Granger cointegration test

Consider two non-stationary time series variables that are integrated of the same order, say order 1, $I(1)$ variables. Following Engle and Granger (1987), two variables, say x and y are said to be cointegrated of order $CI(1,1)$ if there exists a long-run equilibrium relationship between the two integrated variables such that the residuals of the estimated regression are stationary or integrated of order zero, $I(0)$.

The long-run equilibrium relationship is captured by the following regression models:

$$y_t = \alpha_0 + \alpha_1 x_t + w_t \quad (1)$$

$$x_t = \beta_0 + \beta_1 y_t + u_t \quad (2)$$

where x and y are $I(1)$ variables, α_0 , α_1 , β_0 and β_1 are cointegrating parameters, w_t and u_t are OLS residuals which capture divergences between the variables from an assumed equilibrium long-run relationship.

The use of the Engle-Granger (EG) cointegration methodology requires pairwise comparison of two cointegrating regressions because the EG method produces just only one cointegrating vector. We distinguish between the pair of cointegration regressions (1) and (2) above because unlike Johansen cointegration methodology, the Engle-Granger cointegration procedure is sensitive to the choice of dependent variable (see Dickey et al., 1991). Testing for the presence of cointegration in the context of the bivariate Engle-Granger cointegration test is essentially equivalent to testing for the presence of a unit root in the estimated residual series $\{\hat{u}_t\}$ and $\{\hat{w}_t\}$ for the

cointegrating regressions (1) and (2) where the Engle-Granger (EG) tests (which are akin to the standard Dickey-Fuller tests) used for testing the stationarity of the residuals are specified as follows:

$$\Delta\hat{u}_t = \rho_1\hat{u}_{t-1} + \varepsilon_t \quad (3)$$

$$\Delta\hat{w}_t = \rho_2\hat{w}_{t-1} + \varepsilon_t \quad (4)$$

The first difference of the residuals is regressed on the lagged level of the residuals without a constant, where ρ_1 and ρ_2 are parameters of interest representing the slope of each line, $\Delta\hat{u}_t$ and $\Delta\hat{w}_t$ are the first difference of the estimated residual series $\{\hat{u}_t\}$ and $\{\hat{w}_t\}$ respectively, \hat{u}_{t-1} and \hat{w}_{t-1} are the estimated lagged residuals, ε_t and ε_t are error terms which are expected to be serially uncorrelated. Equations (3) and (4) do not include intercept terms because the estimated residual series $\{\hat{u}_t\}$ and $\{\hat{w}_t\}$ are obtained from regression equations (1) and (2) respectively. The EG test requires that error terms be serially uncorrelated. Due to the problem of serial correlation in standard EG test, it is a common practice to use the augmented Engle-Granger (AEG) test which accommodates more lags of the first difference of the residuals to eliminate the serial correlation problem that is associated with standard EG test. The corresponding AEG tests for (3) and (4) are specified as follows:

$$\Delta\hat{u}_t = \rho_1\hat{u}_{t-1} + \sum_{i=0}^p \xi_i \Delta\hat{u}_{t-i} + \varepsilon_t \quad (5)$$

$$\Delta\hat{w}_t = \rho_2\hat{w}_{t-1} + \sum_{j=0}^q \Omega_j \Delta\hat{w}_{t-j} + \varepsilon_t \quad (6)$$

where ρ_1 and ρ_2 are parameters, ξ_i and Ω_j are coefficients of lagged difference of the estimated residuals, $\Delta\hat{u}_t$ and $\Delta\hat{w}_t$ are first difference of the estimated residual series $\{\hat{u}_t\}$ and $\{\hat{w}_t\}$ respectively, \hat{u}_{t-i} and \hat{w}_{t-j} are lags of the estimated residuals, ε_t and ε_t are error terms, p and q are optimal truncation lag parameters to be determined to whiten the error terms. AEG test can be utilized to perform unit root test on the estimated coefficients ρ_1 and ρ_2 individually to establish the existence or non-existence of long-run equilibrium relationship. Any unit root test involving ADF is sensitive to the choice of lag length which is the number of lagged differences with which the regression is augmented. Since AEG test is a modification of ADF test, it also inherits the lag

selection problem that is commonly associated with ADF test due to its sensitivity to the choice of lag length. The main criticism of the Augmented Dickey-Fuller (ADF) test is that the power of the test is very low if the time series under test is nearly non-stationary which implies that the time series is stationary but with a root close to 1 (see Brooks 2002). The focus of our present study is to employ the modified Koyck mean lag approach based on partial correlation criterion (MK-PCC) for lag selection required for the implementation of AEG tests since enough lags need to be chosen for the error terms ε_t and ϵ_t to be serially uncorrelated. In applying the MK-PCC, we consider a distributed lag re-parameterization of the augmented Engle-Granger (AEG) tests as follows:

CASE 1: When y is the dependent variable for the cointegrating regression, we have the following representation:

$$y^{(*)} = \Delta \hat{u}_t - \rho_1 \hat{u}_{t-1} = \sum_{i=0}^p \xi_i \Delta u_{t-i} \tag{7}$$

CASE 2: When x is the dependent variable for the cointegrating regression, we have the following representation:

$$x^{(*)} = \Delta \hat{w}_t - \rho_2 \hat{w}_{t-1} = \sum_{j=0}^q \Omega_j \Delta \hat{w}_{t-j} \tag{8}$$

Using generalized least squares Dickey-Fuller (DF-GLS) test as an alternative unit root test to ADF, we repeat the same distributed lag re-parameterization for the DF-GLS test as follows:

CASE 1: When y is the dependent variable for the cointegrating regression, we have the following representation:

$$y^{(*)} = \Delta \hat{u}_t^d - \rho_1 \hat{u}_t^d = \sum_{i=0}^p \xi_i \Delta u_{t-i}^d \tag{9}$$

CASE 2: When x is the dependent variable for the cointegrating regression, we have the following representation:

$$x^{(*)} = \Delta \hat{w}_t^d - \rho_2 \hat{w}_{t-1}^d = \sum_{j=0}^q \Omega_j \Delta \hat{w}_{t-j}^d \tag{10}$$

Interpretation of notations is the same as earlier given above except that the residual series are subjected to generalized least squares detrending.

3. Modified Koyck mean lag approach based on partial correlation criterion for lag selection (MK-PCC)

Following Koyck (1954) mean lag model, we can assume the Koyck postulations as follows

$$\bar{L}_{(i)} = \frac{R_{(i)}}{1 - R_{(i)}}, \quad i = 1, \dots, 4 \quad (11)$$

where $\bar{L}_{(i)}$ is the mean lag for a particular unit root test, $R_{(i)}$ is the partial correlation coefficient computed for each of the model in equation (7) through equation (10) between $y^{(*)}$ and lagged differences $\Delta y_{t-1}, \Delta y_{t-2}, \dots, \Delta y_{t-12}$ (in case of monthly dataset) and it measures the rate at which $y^{(*)}$ depends on these lagged differences. The main idea of MK-PCC is based on fitting simple linear regression model to the left-hand side of equation (7) through equation (10) to generate the parameters needed and to compute the partial correlation between the parameter on the left-hand side of equation (7) through equation (10) and different choices of lagged differences from the set of lagged differences $\Delta y_{t-1}, \Delta y_{t-2}, \dots, \Delta y_{t-12}$ on the right-hand side of equation (7) through equation (10) while controlling for the effects of other remaining lagged differences. For the first computation we compute partial correlation between $y^{(*)}$ and Δy_{t-1} while controlling for $\Delta y_{t-2}, \dots, \Delta y_{t-12}$. For the second computation we compute partial correlation between $y^{(*)}$ and the first two lagged differences (i.e. $\Delta y_{t-1}, \Delta y_{t-2}$) while controlling for $\Delta y_{t-3}, \dots, \Delta y_{t-12}$ and so on like that. We also repeat the same procedure for other specification of unit root tests as shown above. The partial correlation coefficient denoted by $R_{(i)}$ is computed and adjusted for maximum lag until it gives a values less than 0.3 which is equivalent to lag 0 since for $R_{(i)} < 0.3$, the mean lag will be assumed to be zero since the

mean lag specified by $\bar{L}_{(i)} = \frac{R_{(i)}}{1 - R_{(i)}}$ for which $R_{(i)} < 0.3$ is a fraction not up to

0.5. It should be noted that to have a reasonable mean lag length we expect the absolute value of $R_{(i)}$ to be in the interval $[0.5, 0.999)$ (see Agunloye et al., 2013). The same procedure is repeated for DF-GLS test by fitting simple linear regression model to the left-hand side of equations (9) and (10) to generate the parameters needed and to compute the partial correlation between the parameter on the left-hand side of equation (9) through equation (10) as earlier explained above for ADF test.

As indicated earlier in the introductory part of this paper, the residual-based Engle-Granger cointegration test is very sensitive to the choice of truncation lag parameters p and q . The problem of bias in cointegration is due to misspecification of lag length. Since the Engle-Granger cointegration test is equivalent to testing for the presence of unit root in the estimated residuals from the cointegrating regression it also shares the problem of low power that is commonly associated with unit root test when the estimated residual is closed to being a unit root process but not exactly a unit root process. For the purpose of the present study, we consider a situation when the estimated parameters of interest (i.e. ρ_1 and ρ_2) assume any of the following values: 0.9, 0.95 and 0.999 in equations (7), (8), (9) and (10) respectively. Our choice of these parameter values is informed due to the fact that the power of test for Augmented Dickey-Fuller (ADF) is very low if the process is nearly non-stationary, which means the process is stationary but with a root close to the non-stationary boundary (Brooks 2002).

4. Data description and unit root test

For empirical analysis, we use two sets of data. One real dataset and one simulated dataset. The real dataset are US 3-Month Treasury Bills (USMTB) for short-term money market interest rate series and US 10-Month Government Security (USMGS) for long-term interest rate series. The data cover the period from January 1962 through February 2014 and are obtained from IMF Monthly Bulletin. A total of 626 observations are collected for USMGS and USMTB series respectively.

This paper adopts the residual-based Engle-Granger (EG) cointegration test for empirical analysis. The implementation of EG methodology is carried out in two steps. The first step tests for the order of integration of time series variables. The order of integration of a variable is the number of times a variable is required to be differenced to attain stationarity. A condition applicable to EG test is that the variables entering the cointegrating equation should be integrated of the same order which is assumed to be order 1 in the context of EG test. To test for degree of integration of the USMGS and USMTB series two well-known tests are used in this paper. The first test is the Augmented Dickey-Fuller (ADF) (1984) test and the second test is the generalized least squares Dickey-Fuller (DF-GLS) test introduced by Elliot et al. (1996). The optimal lag length were determined using five lag length selection criteria comprising four conventional criteria and newly introduced criterion called MK-PCC. The results for the unit root tests are presented in tables 1 through table 4 below:

Table 1. Summary of results for ADF Unit root test for both series at level

	AIC	FPE	BIC	HQIC	MK-PCC
USMGS	-1.543344(3)	-1.543344(3)	-1.435517(2)	-1.435517 (3)	-1.341465 (0)
USMTB	-2.577255(3)	-2.577255 (3)	-2.577255 (2)	-2.577255 (3)	-2.299976(0)

The null hypothesis of unit root is rejected if the test statistic is less than the 5% critical value

Table 2. Summary of results for DF-GLS unit root test for both series at level

	AIC	FPE	BIC	HQIC	MK-PCC
USMGS	-0.780907(3)	-0.780907 (3)	-0.684594 (2)	-0.684594 (3)	-0.609149(0)
USMTB	-1.649786(3)	-1.649786 (3)	-1.649786 (2)	-1.649786 (3)	-1.406009 (0)

The null hypothesis of unit root is rejected if the test statistic is less than the 5% critical value

Tables 1 and 2 present the results of unit root tests for the level of the two series under investigation using ADF and DF-GLS tests. The ADF test-statistic under different optimal lag lengths is greater than the critical value at 5% level of significance which is -3.417060. Similarly, the DF-GLS test-statistic under different optimal lag lengths is also greater than the critical value at 5% level of significance which is -2.890000. Consequently, we fail to reject the null hypotheses of unit root for the level of the two series. This implies that each of the series is non-stationary at level. In contrast to standard information criteria which had to fit higher lags such as lag 2 or lag 3 in order to establish non-stationarity of both series at levels, MK-PCC lag selection methodology established non-stationarity of both series without fitting any lag.

Table 3. Summary of results for ADF unit root test for both series after first difference

	AIC	FPE	BIC	HQIC	MK-PCC
$\nabla USMGS$	-12.88040(3)	-16.88413 (3)	-12.17434 (2)	-17.17434 (3)	-16.88413 (0)
$\nabla USMTB$	-17.93451(3)	-17.61138 (3)	-17.93451 (2)	-17.93451 (3)	-17.61138 (0)

The null hypothesis of unit root is rejected if the test statistic is less than the 5% critical value

Table 4. Summary of results for DF-GLS unit root test for both series after first difference

	AIC	FPE	BIC	HQIC	MK-PCC
$\nabla USMGS$	-5.409803(3)	-5.409803 (3)	-6.478566 (2)	-5.409803 (3)	-10.67095 (0)
$\nabla USMTB$	-17.95375(3)	-17.63360(3)	-17.95375(2)	-17.95375 (3)	-17.63360 (0)

The null hypothesis of unit root is rejected if the test statistic is less than the 5% critical value

Tables 3 and 4 present the results of unit root tests for the first difference of the two series under investigation using ADF and DF-GLS tests. The ADF test-statistic under different optimal lag lengths is less than the critical value at 5% level of significance which is -3.417060. Similarly, the DF-GLS test-statistic under different optimal lag lengths is also less than the critical value at 5% level of significance which is -2.890000. Consequently, we reject the null hypotheses of unit root for the two series at first difference. This implies that each series is integrated of order 1 since they become stationary after first difference. The empirical results shown in tables 3 and 4 above show that while stationarity of the first difference of both series was achieved at lag zero under MK-PCC lag selection methodology, the standard information criteria had to fit higher lags such as lag 2 or lag 3 in order to achieve the same results.

5. Engle-Granger cointegration test

We fit autoregressive models of order 1 to 12 to the residuals of the cointegrating regressions and the various optimal lag lengths suggested by different lag selection criteria are presented in brackets in table 5 below. The ADF and DF-GLS unit root tests are performed on the residuals from OLS estimation for USMGS and USMTB pairs. All regressions reported are cointegrated at the 5 per cent level. This suggests that the estimated equations reflect a stable long-run relationships.

Table 5. Engle-Granger cointegration test using ADF test

VARIABLE	AIC	FPE	BIC	HQIC	MK-PCC
USMGS-USMTB RESIDUAL	-3.6199(5)	-3.6199(5)	-3.7089(4)	-3.6199(5)	-3.4822(0)
USMTB-USMGS RESIDUAL	-4.7785(10)	-4.5175(10)	-4.6322(4)	-4.2392(4)	-3.6883(0)

The null hypothesis of “no cointegration” is rejected if the test statistic exceeds the 5% critical value.

Table 5 presents the results of the Engle-Granger cointegration test using ADF unit root test for the stationarity of residuals from each regression equation. For cointegrating regression with USMGS as dependent variable, it is observed that the test statistic for the ADF version of the Augmented Engle-Granger (AEG) test at different optimal lag lengths suggested by conventional lag selection criteria and MK-PCC criterion exceeds the critical value at 5% level of significance. Consequently, we reject the null hypotheses of “no cointegration” at these various optimal lags. This implies that USMGS and USMTB series are cointegrated at these optimal lags. However, for cointegrating regression with USMTB as dependent variable, the test statistic for the ADF version of the Augmented Engle-Granger (AEG) test at different optimal lag lengths suggested conventional lag selection criteria is less than the critical value at 5% level of

significance except for MK-PCC for which the test statistic exceeds the critical value. Hence, we fail to reject the null hypothesis of “no cointegration” under optimal lags suggested by AIC, FPE, BIC and HQIC respectively indicating that USMTB and USMGS are not cointegrated at lag 10 and lag 4 that were suggested by standard information criteria but are cointegrated at lag 0 selected by MK-PCC.

Table 6. Engle-Granger cointegration test using DF-GLS test

VARIABLE	AIC	FPE	BIC	HQIC	MK-PCC
USMGS-USMTB RESIDUAL	-3.6722(5)	-3.6722(5)	-4.0692(4)	-3.6722(5)	-3.4563(0)
USMTB-USMGS RESIDUAL	-4.6524(10)	-4.6967(10)	-4.5326(4)	-4.5326(4)	-3.6005(0)

The null hypothesis of “no cointegration” is rejected if the test statistic exceeds the 5% critical value.

Table 6 presents the results of the Engle-Granger cointegration test using DF-GLS unit root test for the stationarity of residuals from each regression equation. For cointegrating regression with USMGS as dependent variable, it is observed that the test statistic for the DF-GLS version of the Augmented Engle-Granger (AEG) test at different optimal lag lengths suggested by conventional lag selection criteria and MK-PCC criterion exceeds the critical value at 5% level of significance except for BIC which suggested optimal lag 4 for which the test statistic is less than critical value. Consequently, we reject the null hypotheses of “no cointegration” at these various optimal lags. This implies that USMGS and USMTB series are cointegrated under optimal lags suggested by AIC, FPE, HQIC and MK-PCC respectively but they are not cointegrated at lag 4 suggested by BIC. However, for cointegrating regression with USMTB as dependent variable, the test statistic for the DF-GLS version of the Augmented Engle-Granger (AEG) test at different optimal lag lengths suggested conventional lag selection criteria is less than the critical value at 5% level of significance except for MK-PCC for which the test statistic exceeds the critical value. Hence, we fail to reject the null hypothesis of “no cointegration” under the optimal lags suggested by AIC, FPE, BIC and HQIC respectively indicating that USMTB and USMGS are not cointegrated at lag 10 and 4 that were suggested by these standard information criteria but are cointegrated at lag 0 selected by MK-PCC.

5.1. Estimation of Engle-Granger error correction model

Following Engle and Granger (1987), we specify error correction model for the cointegrating relationship between USMGS and USMGTB as follows:

$$\Delta(usmgs)_t = \tau_0 + \sum_{i=1}^{p_1} \gamma_i \Delta(usmgs)_{t-i} + \sum_{j=1}^{q_1} \lambda_j \Delta(usmtb)_{t-j} + \alpha_1 \hat{u}_{t-1} + \varepsilon_t \quad (12)$$

$$\Delta(usmtb)_t = \delta_0 + \sum_{i=1}^{p_2} \phi_i \Delta(usmtb)_{t-i} + \sum_{j=1}^{q_2} \Phi_j \Delta(usmgs)_{t-j} + \alpha_2 \hat{w}_{t-1} + \epsilon_t \quad (13)$$

where α_1 and α_2 are adjustment coefficients, p_1 , q_1 , p_2 and q_2 are the optimal lags required to whiten the error terms in (12) and (13) respectively. In equation (12), USMGS is taken as dependent variable and USMTB is explanatory variable. Similarly in equation (13), USMTB is taken as dependent variable and USMGS is taken as explanatory variable. However, in order for valid inferences to be made from ECM models specified in (12) to (13) above, it is necessary that the coefficients of the lagged residuals represented by α_1 and α_2 , which serve as the “speed of adjustment parameters”, are significant and their coefficients are negative. Mathematically, deviations from long-run equilibrium relationship between two variables can only be corrected if our cointegrating vector is negative. The value of adjustment parameter is a crucial parameter of interest that is expected to be less than 1 in absolute terms to guarantee the stability of the system and for the variables in the long-run relationship to be cointegrated. The number of lags to be included in the ECM equations is determined by the number of lags required to whiten the error terms. The ECM models constructed for USMGS and USMTB series were both valid based on the aforementioned criteria.

5.2. Residual analysis

Prior to estimation of the Engle-Granger error correction model, a crucial issue is whether the error terms are uncorrelated, homoscedastic and normally distributed. Residual analysis was conducted using Breusch-Godfrey LM test for serial correlation, ARCH-LM for heteroskedasticity and Jarque-Bera for normality test. The appropriate number of lags is 2 which is the optimal lag order required to whiten the error term. Bivariate analysis showed that both pairs of USMGS and USGMTB were cointegrated at 5% significance levels. The results of the diagnostic tests on residuals are presented in table 7 below.

Table 7. Summary of results of diagnostic tests on residuals

Tests	Test Statistic	p-value	Conclusion
Jarque-Bera	21.18518	0.000025	Normally distributed
ARCH-LM	972.3744	0.0000	No Heteroskedaticity
Breusch-Godfrey LM test	4194.212	0.0000	No Serial Correlation

The p-values in table above are compared with 0.05 significance level.

Table 8. The Engle-Granger Error Correction Model Estimates for USMGS- USMTB Pair

	Coefficient	t-value	Probability
$(USMGS)_{t-1}$	0.396037	8.42717	0.04700
$(USMGS)_{t-2}$	-0.267194	-5.60852	0.04764
$(USMTB)_{t-1}$	-0.031421	-1.00462*	0.03128
$(USMTB)_{t-2}$	0.050940	1.62607*	0.03133
Residual	-0.023292	-2.53590*	0.00918
Constant	-0.001496	-0.13809	0.01083
R^2	0.151445		
$Adj.R^2$	0.144557		
Sum of Squares Residual	44.95889		
S.E Equation	0.270158		
F-statistic	21.98793		
AIC	0.229978		
BIC	0.272740		

*indicates significance at 5% level

Table 8 presents the empirical result from the short-run dynamics based on the Engle-Granger error correction model when USMGS is taken as dependent variable in the cointegrating regression. In estimating this ECM model, two lags for the explanatory variable were found to be sufficient to whiten the residuals. In the Engle-Granger cointegration methodology, the coefficient of the lagged residual shown in table 8 is of particular interest because it represents the speed of adjustment as well as stability of the system. The absolute value of the coefficient is 0.023292 which is less than 1 indicating that the system is stable. However, the coefficient is quite small which indicates that about 2.3292% of any deviation from the long-run path is corrected within a month which translates into about 27.95% adjustment per year.

Table 9. The Engle-Granger Error Correction Model Estimates for USMTB-USMGS Pair

	Coefficient	t-value	Probability
$(USMTB)_{t-1}$	0.308307	6.53244	0.04720
$(USMTB)_{t-2}$	-0.115712	-2.44775	0.04727
$(USMGS)_{t-1}$	0.321225	4.52965*	0.04092
$(USMGS)_{t-2}$	-0.213002	-2.96287*	0.04189
Residual	-0.023995	-1.78542*	0.01344
Constant	-0.003360	-0.20555	0.01635

Table 9. The Engle-Granger Error Correction Model Estimates for USMTB-USMGS Pair (cont.)

	Coefficient	t-value	Probability
R^2	0.189927		
$Adj.R^2$	0.183352		
Sum of Squares Residual	102.3760		
S.E Equation	0.407670		
F-statistic	28.88513		
AIC	1.052882		
BIC	1.095644		

*indicates significance at 5% level.

Table 9 presents the empirical result from the short-run dynamics based on the Engle-Granger error correction model when USMTB is taken as dependent variable in the cointegrating regression. In estimating this ECM model, two lags for the explanatory variable were also found to be sufficient to whiten the residuals. In the Engle-Granger cointegration methodology, the coefficient of the lagged residual shown in table 9 above is of particular interest because it represents the speed of adjustment as well as stability of the system. The absolute value of the coefficient is 0.023995 which is less than 1 indicating that the system is stable. However, the coefficient is quite small which indicates that about 2.3995% of any deviation from the long-run path is corrected within a month which translates into about 28.79% adjustment per year.

6. Conclusion

This paper examined the problem of lag length selection within the framework of the Engle-Granger cointegration test. We demonstrated that the conventional lag selection criteria such as AIC, FPE, BIC and HQIC standard information criteria have the problem of over-specification of lag length. We introduced a new criterion called the modified Koyck mean lag approach based on partial correlation criterion (MK-PCC) which outperforms conventional standard information criteria by avoiding over-specification of lag length commonly associated with frequently used conventional lag selection criteria.

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