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Methodology of gross value added growth decomposition in the framework of KLEMS productivity accounts for total Polish economy and by voivodship

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Introduction

The name of KLEMS economic productivity accounts originates from symbols traditionally used in formal formulae or from the first letters in English language (K – *Capital*, L – *Labour*, E – *Energy*, M – *Materials*, S – *Services*). It indicates the factors that are referred to in the accounts. These are the so-termed "primary factors", i.e. "capital" and labour", and "secondary factors", which are the components of "intermediate consumption", i.e. "energy", "materials" that include both raw materials and semi-finished products, and "services", being the contributions to firms from outside. The KLEMS productivity accounts present economic processes *ex post,* basically from the supply side point of view. They evolved from a formulation of the neoclassical economic growth theory in the shape of Solow's decomposition from the 1950s. This theory, mostly developed by D. Jorgenson and his associates, is fundamental for two main methodological options presently carried out worldwide. One of them is the OECD methodology, aiming more at maximising international comparability, even at the expense of some compromises against the theory. The second is the KLEMS methodology, being more *de rigueur* with the mainstream theory that is fundamental for this accounting. In the EU KLEMS consortium participate a number of European countries (presently 16) for which growth decomposition, most essential in the KLEMS productivity accounts, is performed but more of them are supposed to be participating (more countries are present already but without the above-mentioned growth decomposition). It has a variation in the form of World KLEMS platform that is intended to become a global one, whereas EU KLEMS is basically a regional initiative. The presented short coverage concerns KLEMS productivity accounts elaborated methodologically and calculated for Poland in the Macroeconomic Studies and Finance Statistics Department of Statistics Poland, and is based on data delivered by Statistics Poland's National Accounts Department, Labour Market Department, and Statistical Office in Bydgoszcz. It also presents some developments of the initial methodology, including a regional decomposition by Polish voivodships.

1. Definition of economic *multifactor productivity* (MFP)

In the KLEMS economic productivity accounts the following formula is used for gross output growth at a given aggregation level *j* for period *t* (Timmer et al. 2007a)[1]:

$$\Delta \ln Y_{jt} = \bar{v}^X_{jt} \Delta \ln X_{jt} + \bar{v}^K_{jt} \Delta \ln K_{jt} + \bar{v}^L_{jt} \Delta \ln L_{jt} + \Delta \ln A^Y_{jt} \quad (1)$$

where Y is gross output, X – intermediate consumption, K – capital, L – labour, whereas A^Y is the *multifactor productivity* (MFP), that can be considered as a variant of TFP used in KLEMS productivity accounts. These values are subscribed to indicate that they concern industries j and periods t. Δ, for all values under this symbol, denotes changes between periods t and $t-1$, usually considered as yearly periods. If the changes are small, as it is usually the case for yearly periods, we can approximate that $\Delta lnx = \Delta x/x$, which allows to interpret these changes as relative changes, e.g. expressed in percentages. $\bar{v}$ with appropriate superscripts and subscripts denote average value shares of the given factors' income (indicated in superscripts as X, K or L) in the gross output, between periods t and $t-1$, that are calculated according to formula $\bar{v} = \left(v_t + v_{(t-1)}\right)/2$ (for simplicity the subscript j present in formula (1) has been omitted here).

The shape of formula (1) results from bestowing to the production function a trans-logarithmic formulation, in order to bring to it an additive property for the relative changes of its independent variables. It is of basic importance, as it eases the use of statistical methods. The growth of A^Y value, i.e. MFP, is calculated as a residual, therefore the formula (1) is always met.

It is quite common in statistics that intermediate consumption deflators, necessary for computing real values, are missing. Moreover, because of large differences between the countries as far as vertical integration of firms is considered, the share of intermediate consumption in gross output varies widely across different countries. Therefore, for international comparisons it is justified to use a decomposition of gross value added (GVA) growth instead[2] since it does not include intermediate consumption. So, on the EU KLEMS platform it is the GVA growth decomposition which is carried out extensively. A similar trans-log function is used here, but without the intermediate consumption X as a component "factor":

$$\Delta \ln V_{jt} = \bar{w}^K_{jt} \Delta \ln K_{jt} + \bar{w}^L_{jt} \Delta \ln L_{jt} + \Delta \ln A^V_{jt} \quad (2)$$

where V stands for GVA, and the other symbols (with appropriate superscripts and subscripts) have the same meaning as in formula (1) but, with the exception of capital K and labour L, take different values. It

[1] KLEMS productivity accounts are based mainly on the works of Dale Jorgenson and associates (1963, 1967, 1987, 1989, 2005). This methodology has been adapted to Polish conditions and further developed (see Kotlewski & Błażej, 2018, 2020 and Kotlewski, 2021).

[2] Lastly, however, the gross output growth decomposition has been performed in Statistics Poland, which nevertheless has an additional analytical merit.

is important to note that the analogical average shares $\bar{w}$ (of production-factor remunerations in the GVA) are not identical to average shares $\bar{v}$ from formula (1) (they are calculated in a similar way as arithmetic averages, however). Also, the contribution of MFP from the decomposition of GVA V growth is not identical in relative terms with the contribution of MFP from the decomposition of gross output Y growth, although its absolute growth value in the ideal case where there would be no variability associated with intermediate consumption should be identical.

2. Gross output, intermediate consumption and GVA accounts

The gross output growth in industry j in period t, i.e. exactly the left-hand side of the equation (1), is defined as the sum of weighted contributions of the growth rates of individual products i to the overall growth of the entire industry j in value terms:

$$\Delta \ln Y_{jt} = \sum_i \bar{v}^{Y}_{ijt} \Delta \ln Y_{it} \tag{3}$$

where $\bar{v}$ with appropriate superscript and subscripts stands for average value shares of the individual products i of the given industry j in gross output Y of the entire industry j, between the periods t and $t-1$ (they are calculated similarly as above, according to formula $\bar{v} = (v_t + v_{(t-1)})/2$).

The growth of intermediate consumption in the given industry can be defined by the formula:

$$\Delta \ln X_{jt} = \sum_i \bar{v}^{X}_{ijt} \Delta \ln X_{it} \tag{4}$$

where subscript i stands for individual products as above, whereas $\bar{v}$ with appropriate superscript and subscripts stand for average value shares of the individual products i of the given industry j in intermediate consumption X of industry j, between the periods t and $t-1$, calculated in a similar way to the shares in formula (3).

Similarly, the growth of gross value added (GVA) can be defined by the formula:

$$\Delta \ln V_{jt} = \sum_i \bar{w}^{V}_{ijt} \Delta \ln V_{it} \tag{5}$$

where V stands for GVA, and the superscript V for w indicates that the shares are in the GVA of entire industry j.

To calculate both the gross output and the intermediate consumption supply and use tables (SUT) can be used as second best, instead of symmetric input-output tables (IOT), which are available only periodically – on the condition, however, that they are consistent with SNA[3] and ESA[4] systems and their updates. The

[3] *System of National Accounts.*
[4] *European System of Accounts.*

Eurostat transmission tables are consistent with ESA, and the above-mentioned SUT tables. Therefore, they can also be used in KLEMS accounting as data source.

3. Labour factor accounts

Similarly to the growths of gross output, of intermediate consumption and of GVA, can be defined the growth of labour factor at industry j level, according to the formula:

$$\Delta \, ln \, L_{jt} = \sum_l \bar{v}_{ljt} \, \Delta \, ln \, H_{ljt} \tag{6}$$

where L denotes labour factor services, l – the different types of labour, v with appropriate subscripts – average income value shares of the different labour types l from two periods t and $t-1$ (calculated in a similar way to previously mentioned shares as arithmetic averages), and H with appropriate subscripts – the number of hours worked in the given labour type l in period t. The labour types are distinguished according to sexes, three age groups and three education attainment levels. It is assumed here that the values of labour services of all types l are proportional to the hours worked in these labour types and that the employees of each labour type are remunerated according to their marginal productivity, which translates into their compensation. Hence, we have the value shares of labour types, as their compensation shares in the overall labour compensation of industry j. The labour-type shares $\bar{v}_{ljt}$ in industries j are calculated similarly to products i shares in industries j in previous formulae (3), (4) and (5). Thus we have 18 (according to sexes, three age groups and three education attainment levels) types of labour l, i.e. the formula (6) can be specified as:

$$\Delta \, ln \, L_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta lnH_{ljt} \tag{7}$$

The labour factor understood in this way (as labour services) can be decomposed into one component consisting of sheer growth of the number of hours worked, and the other component related to labour quality understood as labour composition LC change, and the appropriate formula for the growth of this labour quality is:

$$\Delta lnLC_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta lnH_{ljt} - \Delta lnH_{jt} \tag{8}$$

However, the labour factor can also be extended by remuneration growth effect. Then the formula for labour quality SC understood in this way is:

$$\Delta lnSC_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta lnW_{ljt} - \sum_{l=1}^{18} \bar{v}_{ljt} \Delta lnH_{ljt} \tag{9}$$

where W are compensations of the given labour types l, in the given industry j, in period t, which should be consistent with:

$$\Delta lnSC_{jt} = \Delta lnW_{jt} - \Delta lnL_{jt} \quad (10)$$

basing on formula (6) and the observation that the first term on the right-hand side of equation (9) is nothing else than the sum of weighted compensation growths.

Labour quality on the EU KLEMS platform (also on the World KLEMS one) is understood as labour composition, but in Statistics Poland the calculations have been done in both ways.

When addressing these theoretical issues, the problem is that often the labour factor is measured in inappropriate units, i.e. employees instead of hours. Moreover, these hours are often hours paid instead of hours worked that are considered to be appropriate in KLEMS productivity accounting.

The phenomenon of self-employment is also an issue to be addressed. Some data adjustments were therefore necessary. The values of the labour factor were adjusted for the self-employment, thanks to the assumption that hours worked of the self-employed are remunerated in the same way as hours worked of the contractually employed according to NACE activities respectively. This assumption is generally adopted by all countries performing KLEMS productivity accounts.

4. Capital factor accounts

The growth of the capital factor can be similarly defined according to formula:

$$\Delta \ ln \ K_{jt} = \sum_k \bar{v}_{kjt} \, \Delta \ ln \ a_{kjt} \quad (11)$$

In this formula symbols K for capital factor services and a for individual assets have replaced L and H used in the formula (6) for the labour factor. The average shares $\bar{v}$ from two periods are calculated similarly for individual asset types k, i.e. as shares in the overall capital remuneration of industry j. Therefore, the growth of capital factor is defined in the KLEMS productivity accounts as the growth of capital services, not as the growth of capital assets[5].

A division of capital assets into 9 types has been adopted in KLEMS accounting[6]:

1) dwellings,
2) other structures and buildings,
3) transport equipment,

[5] The arbitrage equation derived from the neo-classical theory of investment is used here, as introduced by Jorgenson (1963) and Jorgenson & Griliches (1967). It is assumed that the relative growths of "capital services" are equal to the relative growths of capital asset stocks (although their absolute growths are not) at the lowest adopted aggregation level. From it arises the necessity to operate only with relative growths in the entire KLEMS accounts.

[6] In some versions of KLEMS accounts, performed in some countries, a division into more asset categories is sometimes adopted.

4) other machinery and equipment,
5) computing equipment,
6) communication equipment,
7) cultivated assets,
8) intellectual property products, and
9) software.

Three types of capital, i.e. 5) computing equipment, 6) communication equipment, and 9) software are not being extracted in the Polish conditions from other aggregates in source data, and therefore this additional operation had to be carried out – data structures contained in supply and use tables (SUT) were used for this purpose. After their extraction and the application of formula (11), these three categories are aggregated into the so-termed ICT capital, whereas the remaining capital items are aggregated in the so-termed non-ICT capital. In this way the capital factor is divided into two sub-factors:

$$\overline{w}_{jt}^{K}\Delta \ln K_{jt} = \overline{w}_{jt}^{KIT}\Delta \ln KIT_{jt} + \overline{w}_{jt}^{KNIT}\Delta \ln KNIT_{jt} \tag{12}$$

where KIT denotes the ICT capital and $KNIT$ – the non-ICT capital. The extraction of the ICT capital into a separate sub-factor was based on the conviction that it is especially important for the KLEMS accounts, but this fact was confirmed only for some countries[7].

There are controversies whether dwellings should be treated as production capital in these accounts. Because of Polish dwelling market specificities, at first it has been decided not to consider the residential capital. But from the point of view of international comparisons it is needed because the accounts for the above-mentioned European countries belonging to the EU KLEMS system are carried out together with the dwellings. In Statistics Poland the calculations have been done for both of these assumptions and the results are published accordingly. Therefore, since we have two ways of calculating labour quality (*LC* or *SC* as above mentioned) and two ways of calculating capital values (with and without residential capital), the result data are published for the four possible variants indicated as A, B, C and D, of which the B version is closest to the way of making the calculation of EU KLEMS platform and therefore it is the one that should be used in international comparisons.

The basic way to assess the stock of capital assets is to assume some initial capital stock in some base year and to calculate the changes by subtracting capital depreciation and adding up investments[8] according to the *perpetual inventory method (PIM)*. The appropriate data concerning capital asset stocks are available according to lower aggregations than that required in the EU KLEMS accounts. Because all 34 EU KLEMS aggregations are at higher or at the same levels as NACE aggregations (divisions), it was possible to use the data from the National Accounts by summing them up to EU KLEMS aggregations. In the EU KLEMS methodology the capital depreciation is calculated according to the USA capital

[7] In the KLEMS accounts done in the USA it is not practiced. Rather the intensity of ICT use in different industries is being studied.
[8] In this elaboration we differentiate linear (arithmetic) depreciation provisions practiced by the accountants and the capital depreciation which in the KLEMS methodology is non-linear (geometric).

depreciation indices (geometrical), which can be used for all countries and are used by EU KLEMS participants. However, some of these values are provided as value ranges, so for individual countries it remains necessary to individually assess them, which is usually already provided by the National Accounts.

There are some problems with international comparability of some types of capital. In different countries are used different definitions (narrow and wide) of IT and CT industries. The initial capital stocks are also an issue for international comparability, as they are differently assessed in different countries and for different base years. There is also the problem of leased capital services. The services of capital that is not owned by the user are usually considered as a kind of intermediate consumption, instead of being considered as a component of the capital factor (at the same time as above mentioned, the individual countries are different as far as vertical integration of firms is considered, therefore the intermediate consumption is hidden within the vertically integrated firms to a different degree in different countries). Moreover, sometimes there is the issue of public participation in infrastructural investments, which should be treated in the same way as private investments. And in addition, the countries treat differently the price deflation which is present and specific for ICT products. Therefore, the price indices applied by different countries for the ICT industry are very different, and it is so even for countries with similar economies as in Europe.

5. Calculation specificity in KLEMS accounts for Poland

It seems justified to carry out KLEMS productivity accounting for Poland basing on the assumptions as close as possible to those applied by the EU KLEMS countries. The main approach is to apply initially only a GVA growth decomposition, which eases international comparisons (although later the decomposition of gross output growth has been carried out as well in Statistics Poland[9] – it is published in parallel on this internet site).

For the labour factor, the data from a representative survey (performed by the Statistical Office in Bydgoszcz on Z-12 forms) are available for the even years 2004, 2006, 2008, 2010, 2012, 2014, 2016, 2018, and 2020, etc. Therefore, they are sufficient to carry out the KLEMS productivity accounts for Poland flung back to 2005. For the uneven years linear interpolation has been applied. For 2004 these data concern the number of full-time employees, the average remunerations by hour worked in the nominal time and in the overtime of full-time employees in the entire year, and the number of hours worked of these full-time employees. From 2006 onward, the data concern employees, not only full-time employees. Because these data are adjusted upward by a structure concerning the entire labour market, the possible errors here become negligible.

The above-mentioned data for 2004 and 2006 are in the NACE 1 classification system. From 2008 onward the data are in the NACE 2 classification system, but the data for 2008 have been converted also into NACE 1 (by the Department of Labour Market of Statistics Poland). In EU KLEMS a fixed correspondence between NACE 1 and NACE 2 is applied for the labour factor and the same has been practiced in the KLEMS accounts

[9] As above mentioned, it has some additional analytical value.

for Poland (subdivision into 14 sections and section groups). The labour factor has been assessed, considering the self-employment, for the total number of hours worked and total compensation of this factor.

For the capital factor, the basic operation was to extract the above-mentioned three types of ICT capital before aggregating them into a single category of ICT capital. This was done thanks to supply and use tables (SUT), where there are figures in the column „gross fixed capital formation" for all three types of ICT capital. These three figures were distributed proportionally to the horizontal structure of software services taken from the same supply and use tables, which was transposed in a vertical position and aggregated into 34 vertically arranged KLEMS aggregations, basing on the assumption that at the industry aggregation level the values of software services are quite proportional to the values of the three categories of ICT capital. The capital stocks calculated in this way were then extracted from aggregates where they were contained before. Because the supply and use tables are elaborated in two different classification systems NACE 1 and NACE 2 and they will not be recalculated, it was necessary to use the above-mentioned correspondence of 14 sections and section groups applied for the labour factor. For the latest years in the performed KLEMS accounting the last available supply and use tables were applied, since they are done with a greater delay in comparison to other data sources. The practice to use supply and use tables from previous years is quite common since the structures of data in these tables change very slowly.

One expected problem in KLEMS productivity accounts was the passing from ESA'95 system to ESA 2010 system, because not all data have been already recalculated into the new system, and some data shall never be recalculated (e.g., it concerns supply and use tables from before 2010). Therefore, there was sometimes the need to use mixed data (which is applied only for the ICT capital extraction). To test whether this is acceptable, an analysis was carried out for differences in places were the data are available in both systems, i.e. by comparing the differences between capital stock growth rates from both classifications. The results of this analysis have shown that the inconsistencies are negligible from the point of view of KLEMS productivity accounts requirements.

6. Calculation at regional level by Polish voivodships

Further methodological work on the KLEMS productivity accounts showed that the available statistical data now make it possible to perform a decomposition not only for the aggregate Polish economy, but also for the Polish voivodeships. However, the fulfilment of this KLEMS regional productivity accounts involved additional and very substantial methodological and computational work (Kotlewski, 2020).

These calculations were preceded by the performance of a more basic multivariant factor decomposition:

- Decomposition of GVA growth into contributions of factor remuneration for employed persons (I),
- Decomposition of GVA growth into contributions of factor stocks and TFP for employed persons (II),
- Decomposition of GVA growth into contributions of factor stocks and TFP for working persons (III),
- Decomposition of GVA growth into contributions of factor remuneration for working persons (IV).

Among the four variants above, the third item (III) represents a fully performed decomposition according to Robert Solow's basic formula. The final fourth variant (IV) was performed to complete the entire system of multivariant factor decomposition. This decomposition was performed, for the total Polish economy, by groups of sections and sections of NACE (11 aggregations), by voivodships (16 aggregations) and simultaneously by groups of sections and sections of NACE, and by voivodeships (11 x 16 lowest aggregations). The multivariant nature of the decomposition proved to be its advantage, as it allows additional comparative analyses to be carried out.

The data processing methods developed during the implementation of the multivariant decomposition were, after their considerable development, further applied in the implementation of the proper KLEMS productivity accounts by voivodship, which is also based on the idea of factor decomposition initiated by Solow but developed by Jorgenson and his associates[10]. The data structures of the Bank of Local Data (BDL) published by Statistics Poland were used to distribute some data by voivodship (GVA and its components). Data by voivodship on the structure of the labour factor came from a sample survey conducted on the Z-12 form and were prepared for the KLEMS accounts by the Statistical Office in Bydgoszcz. Data related to the capital factor were assessed by voivodeship by means of using the structure of data on fixed assets by NACE sections and voivodships provided by the National Accounts Department of Statistics Poland.

7. Publication of result data

Because of the strong methodological and thematic relationship of the multivariant factor decomposition with the proper KLEMS productivity accounts and because of its additional analytical value due to its multivariant nature, it is published (updated) alongside the proper KLEMS productivity account.

When publishing the result data of the proper KLEMS productivity accounts for the Polish economy and by voivodship, a specific solution was applied. The data are presented as contributions to the growth rates of aggregate GVA or as contributions to sectoral GVA growth rates. The first way of presenting the data (in tables marked with symbols A, B, C and D) has the advantage that it allows for assessing the weight of the contributions related to individual sectors to the growth rate of aggregate GVA (at total Polish economy and voivodships levels) and is a solution specific to the KLEMS productivity accounts performed for the Polish economy. The second way (in the tables marked with symbols A', B', C' and D' below in the same Excel sheets) practised more universally makes it easier to make comparisons between different sectors on the same charts.

[10] Details of this issue are described at length in: Kotlewski (2020).

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