TECHNICAL DESCRIPTION

Structure of the database with the results of the multivariant factor decomposition of GVA

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The database with the results of the multivariant factor decomposition of GVA consists of four result files prepared in Excel format. Each file consists of four worksheets. Each of the four worksheets in Excel files I and IV[1], which relate to the decomposition of the gross value added (GVA) growth rates into factor remuneration contributions, consists of tables grouped into 24 columns. Excel files II and III, which relate to the decomposition of GVA growth rates into factor resources (or stocks) contributions and TFP (total factor productivity), consist of tables grouped into 19 columns.

Figure 1. Worksheet for the factor decomposition of GVA growth rate into factor remuneration contributions in Excel files I and IV for employed persons (Z) and for working persons (P).

Each of the four worksheets of Excel file I for employed persons and the four worksheets of Excel file IV for working persons concerning the decomposition of GVA growth rate into factor remuneration contributions have the structure as presented in Figure 1. The colours make it easier to navigate through the extensive worksheets. The meaning of the abbreviations in the column group headings in the diagram is as follows:

- UP – labour share;
- RK – gross capital profitability;

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[1] Numbering was adopted in the order of their execution.

- Dec ΔGVA – decomposition of aggregate GVA growth rate into contributions of production factor remuneration;
- Dec ΔGVA/ZvP – decomposition of gross GVA growth rate per employed person (Z) in File I or per working person (P) in File IV into contributions of production factor remuneration;
- Dec O GVA/ZvP – decomposition of deviations from the national average GVA per employed person (Z) in file I or per working person (P) in file IV into contributions of production factor remuneration;
- Dec ΔO GVA/ZvP – decomposition of changes in deviations from the national average GVA per employed person (Z) in file I or per working person (P) in file IV into contributions of production factor remuneration.

These titles are written in full in the column group headings in Excel files I and IV. Each column with a series of tables ends with a bold highlighted table with a summary of the column (the rectangles with the word 'sums' in Figure 1). The individual table headings make it additionally easier for the user to remain aware which part of the worksheet they are browsing.

In the first worksheet, the heads of the tables show the voivodships, and the sides show the subsequent years. The tables are arranged vertically according to subsequent sections or statistically available (for the needs of the project) groups of sections. In the second worksheet, sections or statistically available groups of sections are presented in the heads of the tables and the subsequent years are on the sides. The tables are arranged vertically, according to voivodships. In principle, this worksheet is redundant in relation to the first one; however, using it when plotting sections (as for voivodships in time) will make the work much easier (as moving to distant places in the worksheet becomes unnecessary). In the third sheet, voivodships were placed in the heads, while the sections or statistically available groups of sections were placed on the sides. The tables are arranged vertically by successive years. This worksheet allows the performance of in-depth analyses for specific individual years on graphs, in which voivodships are presented on the horizontal axis instead of the consecutive years. In the fourth worksheet, the tables from the third worksheet are transposed (sections are placed in the heads instead of provinces, and vice versa on the sides). Although this worksheet is redundant in relation to the third one, it has been left in the database in case the user prefers this kind of data arrangement. All the data collected in the four sheets are identical, but distributed differently. In the first and second worksheet, time charts (thumbnails) have been created inside individual tables, under each column containing the data to facilitate a quick interpretation of the figures in the tables.

Due to the fact that the calculation of growth rates requires data from the previous year, some empty tables or empty rows can be found in the sheets for the year 2000. The tables for 2008 have been repeated because in some cases the result data are different. It depends on whether the source data come from the NACE Rev.1.1 or NACE Rev. 2 classification system, which are both available for the year 2008. In order to calculate phenomena occurring up to 2008, one should use the tables located above, which sometimes uses source data from the NACE Rev. 1.1 system[2]. In turn, in order to calculate e.g. growth rates for 2009, one should use the tables located below, which are fully based on the NACE Rev. 2 classification system. It should be noted that

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[2] It refers to those source data which have not been systemically converted by Statistics Poland into NACE Rev. 2.

at the level of individual cells, the differences between the upper and lower tables for 2008 may be sometimes significant.

The structure of the result tables for the decomposition taking into account the extraction of TFP looks slightly different. Excel files II and III concern the decomposition into contributions of factor resources (stocks) and the TFP contribution; file II is for the employed and file III for working persons. Both of these files consist of four worksheets. In each worksheet, the tables have been grouped into 19 columns (there are slightly fewer than in files I and IV, as it was unnecessary to duplicate the first four columns from the Excel files on the decomposition into factor remuneration; moreover, as formally explained in the monograph[3], some of the final mathematical formulae have become shorter).

Each of the four worksheets of the Excel file for employed persons (II) and the four worksheets of the Excel file for working persons (III), concerning the decomposition into contributions of factor resources (stocks) and TFP, has the same structure as in Figure 2. Here too, colours have been used to make the extensive worksheets easier to navigate. The abbreviations in the heads are explained below:

- Dec ΔGVA – decomposition of aggregate GVA growth rate into contributions of factor resources (stocks) and TFP;
- Dec ΔGVA/ZvP – decomposition of GVA growth rate per employed person (Z) in file II or per working person (P) in file III into contributions of capital and TFP;
- Dec O GVA/ZvP – decomposition of deviations from national average GVA per employed person (Z) in File II or per working person (P) in File III into contributions of capital and TFP;
- Dec ΔO GVA/ZvP – decomposition of changes in deviations from national average GVA per employed person (Z) in File II or per working person (P) in File III into contributions of capital and TFP.

In the above operations, the labour input gets shortened in the decomposition formulae per employee (Z) in File II or per worker (P) in File III. Therefore, there are only two contributions of capital and TFP[4].

Figure 2. Worksheet for the factor decomposition of GVA growth rate into factor resources (stocks) contributions and TFP contribution in Excel files II for employed persons (Z) and III for working persons (P).
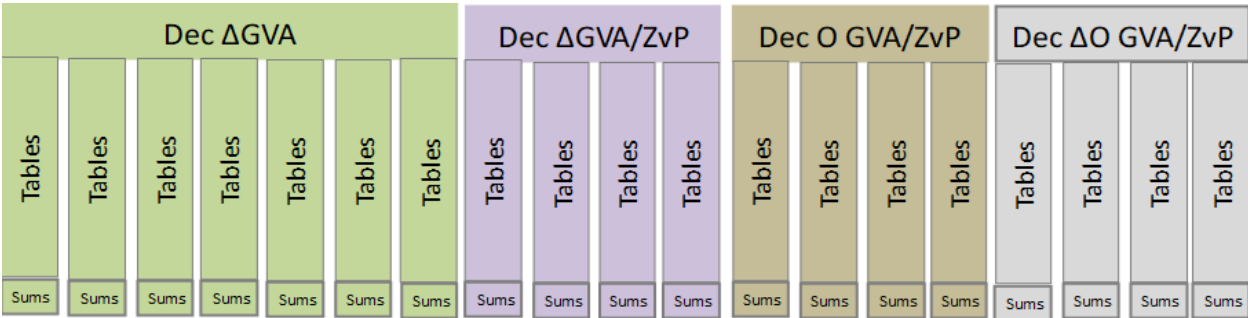
The titles are written in full in the attached Excel files. Each column with a series of tables ends with a table which contains a summary of the column (a rectangle labelled 'sums' in Figure 2). The structure of Excel files
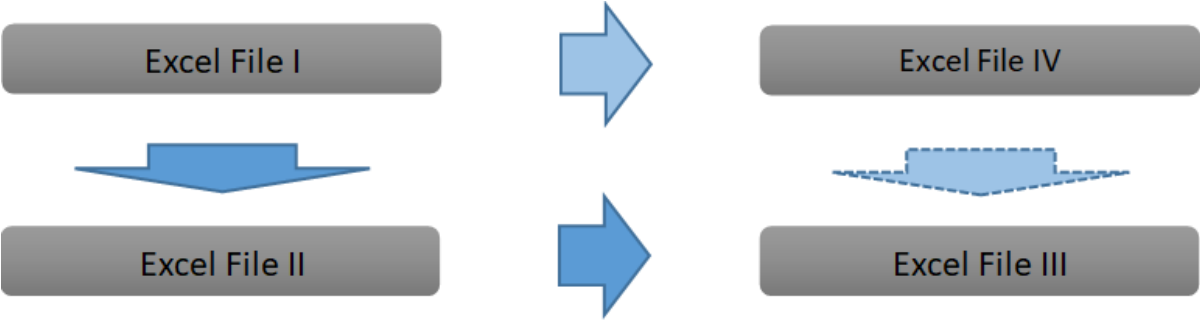
II and III for the decomposition into factor resources contributions and the TFP contribution is otherwise similar to the two previously described Excel files (I and IV).

The diagram below clearly illustrates how the four Excel files (I, II, III and IV), each with four worksheets of result data arranged in groups of columns, relate to each other.

Figure 3. Main project products in the framework of TAOP II

As Figure 3 demonstrates, after the completion of the first stage (file I, related to the decomposition into factor remuneration contributions, for the labour factor considered as employed persons), stage two (file II, related to the decomposition into factor resources (stocks) contributions and TFP contribution) and stage three (file III, related to the conversion of stage two into accounting made for the labour factor considered as working persons) were completed. It is also possible to convert stage one (file I) into accounting made for the labour factor understood as working persons (file IV), which is indicated by a lighter horizontal arrow in Figure 3. Theoretically, following this route (which in Figure 3 is indicated by a vertical arrow with a dashed outline), stage three could also be reached. Table 1 is a collection of 86 computing operations (final algorithms) corresponding to the 86 groups of columns from Figures 1 and 2 above. The arrangement of Table 1 is conformable with Figure 3.

Table 1. List of computational operations of the multivariant factor decomposition of the GVA growth rate

Accounting for the labour factor understood as employed persons	Accounting for the labour factor understood as working persons
Accounting for factor remuneration (without a TFP extraction)	
Excel file: *I – Decomposition of GVA growth rate into factor remuneration contributions for employed persons (Z)*	**Excel file:** *IV – Decomposition of GVA growth rate into factor remuneration contributions for working persons (P)*

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Labour share

1) Labour remuneration share (UWP_Z) in the GVA (WDB):

$UWP_Z = WP_Z/WDB$ [percentages].

2) Change in the labour remuneration share (ΔUWP_Z) in the GVA (WDB):

$\Delta UWP_Z = WP_Z/WDB - WP_{Z(-1)}/WDB_{(-1)}$ [percentage points].

Gross capital profitability

3) Gross capital profitability (RK_Z) [ratio of residually calculated capital remuneration (WK_Z) to the capital stock value (K)]: $RK_Z = WK_Z/K$ [percentages].

4) Change in the gross capital profitability: $\Delta RK_Z = WK_Z/K - WK_{Z(-1)}/K_{(-1)}$ [percentage points].

Decomposition of the aggregate GVA growth rate into contributions of production factor remuneration

5) Real GVA growth rate: $\delta wdb = \Delta WDB/WDB_{(-1)}$ [percentages], where $\Delta WDB = WDB - WDB_{(-1)}$; **WDB** is in constant prices and $WDB_{(-1)}$ is in current prices;

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Labour share

1) Labour remuneration share (UWP_P) in the GVA (WDB):

$UWP_P = WP_P/WDB$ [percentages].

2) Change in the labour remuneration share (ΔUWP_P) in the GVA (WDB):

$\Delta UWP_P = WP_P/WDB - WP_{P(-1)}/WDB_{(-1)}$ [percentage points].

Gross capital profitability

3) Gross capital profitability (RK_P) [ratio of residually calculated capital remuneration (WK_P) to the capital stock value (K)]: $RK_P = WK_P/K$ [percentages];

4) Change in the gross capital profitability: $\Delta RK_P = WK_P/K - WK_{P(-1)}/K_{(-1)}$ [percentage points].

Decomposition of the aggregate GVA growth rate into contributions of production factor remuneration

5) Real GVA growth rate: $\delta wdb = \Delta WDB/WDB_{(-1)}$ [percentages], where $\Delta WDB = WDB - WDB_{(-1)}$; **WDB** is in constant prices and $WDB_{(-1)}$ is in current prices;

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6) Real labour remuneration growth rate: $\delta wp_z = \Delta WP_Z/WP_{Z(-1)}$ [percentages], where $\Delta WP_Z = WP_Z - WP_{Z(-1)}$, WP_Z is in constant prices, $WP_{Z(-1)}$ is in current prices.

7) Real capital remuneration growth rate: $\delta wk_z = \Delta WK_Z/WK_{Z(-1)}$ [percentages], where: $\Delta WK_Z = WK_Z - WK_{Z(-1)}$, WK_Z is in constant prices and $WK_{Z(-1)}$ in current prices

8) Contribution of labour remuneration in the GVA growth: $\alpha_z*\delta wp_z = \delta wp_z*(UWP_Z+UWP_{Z(-1)})/2$ [percentage points].

9) Contribution of capital remuneration in GVA growth, theoretically: $\beta_z*\delta wk_z = \delta wk_z*[(1-UWP_Z)+(1-UWP_{Z(-1)})]/2$ [percentage points], in practice calculated residually based on the assumption that: $\beta_z*\delta wk_z = \delta wdb - \alpha_z*\delta wp_z$.

<u>Equation 5)=8)+9) is fulfilled.</u>

<u>Decomposition of the GVA growth rate per employed person into contributions of production factor remuneration</u>

10) Real GVA growth rate per employed person (Z): $\delta wdb/z = \Delta(WDB/Z)/(WDB_{(-1)}/Z_{(-1)})$ [percentages], $\Delta(WDB/Z) = WDB/Z - WDB_{(-1)}/Z_{(-1)}$, where WDB is in constant prices and $WDB_{(-1)}$ is in current prices.

11) Real labour remuneration growth rate per employed person (Z): $\delta wp_z/z = \Delta(WP_Z/Z)/(WP_{Z(-1)}/Z_{(-1)})$ [percentages],

6) Real labour remuneration growth rate: $\delta wp_p = \Delta WP_P/WP_{P(-1)}$ [percentages], where: $\Delta WP_P = WP_P - WP_{P(-1)}$, WP_P is in constant prices, $WP_{P(-1)}$ is in current prices.

7) Real capital remuneration growth rate: $\delta wk_p = \Delta WK_P/WK_{P(-1)}$ [percentages], where: $\Delta WK_P = WK_P - WK_{P(-1)}$, WK_P is in constant prices and $WK_{P(-1)}$ in current prices.

8) Contribution of labour remuneration in the GVA growth: $\alpha_p*\delta wp_p = \delta wp_p*(UWP_P+UWP_{P(-1)})/2$ [percentage points].

9) Contribution of capital remuneration in GVA growth, theoretically: $\beta_p*\delta wk_p = \delta wk_p*[(1-UWP_P)+(1-UWP_{P(-1)})]/2$ [percentage points], in practice calculated residually based on the assumption that: $\beta_p*\delta wk_p = \delta wdb - \alpha_p*\delta wp_p$.

<u>Equation 5)=8)+9) is fulfilled.</u>

<u>Decomposition of the GVA growth rate per working person into contributions of production factor remuneration</u>

10) Real GVA growth rate per working person (P): $\delta wdb/p = \Delta(WDB/P)/(WDB_{(-1)}/P_{(-1)})$ [percentages], $\Delta(WDB/P) = WDB/P - WDB_{(-1)}/P_{(-1)}$, where WDB is in constant prices and $WDB_{(-1)}$ is in current prices.

11) Real labour remuneration growth rate per working person (P): $\delta wp_p/p = \Delta(WP_P/P)/(WP_{P(-1)}/P_{(-1)})$ [percentages],

$\Delta(WP_Z/Z) = WP_Z/Z - WP_{Z(-1)}/Z_{(-1)}$, where WP_Z is in constant prices and $WP_{Z(-1)}$ is in current prices.

12) Real capital remuneration growth rate per employed person:
$\delta wk_Z/z = \Delta(WK_Z/Z)/(WK_{Z(-1)}/Z_{(-1)})$ [percentages],
$\Delta(WK_Z/Z) = WK_Z/Z - WK_{Z(-1)}/Z_{(-1)}$, where WK_Z is in constant prices and $WK_{Z(-1)}$ is in current prices.

13) Contribution of labour remuneration in the GVA growth per employed person: $\alpha_z \cdot \delta wp_Z/z = (\delta wp_Z/z)\cdot(UWP_Z+UWP_{Z(-1)})/2$ [percentage points].

14) Contribution of capital remuneration in the GVA growth per employed person, theoretically: $\beta_z \cdot \delta wk_Z/z = (\delta wk_Z/z)\cdot[(1-UWP_Z)+(1-UWP_{Z(-1)})]/2$ [percentage points], in practice calculated residually based on the assumption that $\beta_z \cdot \delta wk_Z/z = \delta wdb/z - \alpha_z \cdot \delta wp_Z/z$.

Equation 10)=13)+14) is fulfilled.

Decomposition of the deviations from the national average GVA per employed person into contributions of factor remuneration

15) Deviation (O) from the national average GVA per employed person:
$O(WDB/Z) = (WDB_i/Z_i - WDB/Z)/(WDB/Z)$ [percentages], where i is the variable for a given voivodship, for a given section or for a given voivodship and section.

16) Deviation from the national average labour remuneration per employed person: $O(WP_Z/Z) = (WP_{Z,i}/Z_i - WP_Z/Z)/(WP_Z/Z)$ [percentages].

$\Delta(WP_P/P) = WP_P/P - WP_{P(-1)}/P_{(-1)}$, where WP_P is in constant prices and $WP_{P(-1)}$ is in current prices.

12) Real capital remuneration growth rate per working person:
$\delta wk_P/p = \Delta(WK_P/P)/(WK_{P(-1)}/P_{(-1)})$ [percentages],
$\Delta(WK_P/P) = WK_P/P - WK_{P(-1)}/P_{(-1)}$, where WK_P is in constant prices and $WK_{P(-1)}$ is in current prices.

13) Contribution of labour remuneration in the GVA growth per working person: $\alpha_p \cdot \delta wp_P/p = (\delta wp_P/p)\cdot(UWP_P+UWP_{P(-1)})/2$ [percentage points].

14) Contribution of capital remuneration in the GVA growth per working person, theoretically: $\beta_p \cdot \delta wk_P/p = (\delta wk_P/p)\cdot[(1-UWP_P)+(1-UWP_{P(-1)})]/2$ [percentage points], in practice calculated residually based on the assumption that $\beta_p \cdot \delta wk_P/p = \delta wdb/p - \alpha_p \cdot \delta wp_P/P$.

Equation 10)=13)+14) is fulfilled.

Decomposition of the deviations from the national average GVA per working person into contributions of factor remuneration

15) Deviation (O) from the national average GVA per working person:
$O(WDB/P) = (WDB_i/P_i - WDB/P)/(WDB/P)$ [percentages], where i is the variable for a given voivodship, for a given section or for a given voivodship and section.

16) Deviation from the national average labour remuneration per working person: $O(WP_P/P) = (WP_{P,i}/P_i - WP_P/P)/(WP_P/P)$ [percentages].

17) Deviation from the national average capital remuneration per employed person: $O(WK_Z/Z) = (WK_{Z,i}/Z_i - WK_Z/Z)/(WK_Z/Z)$ [percentages].

18) Contribution of labour remuneration in the deviation of the GVA from the national average per employed person: $\alpha_z*O(WP_Z/Z) = O(WP_Z/Z)*UWP_Z$ [percentage points]; α_z is not calculated intertemporally.

19) Contribution of capital remuneration in the deviation of the GVA from the national average per employed person, theoretically: $\beta_z*O(WK_Z/Z) = O(WK_Z/Z)*(1-UWP_Z)$ [percentage points]; β_z is not calculated intertemporally; in practice, calculated residually based on the assumption that: $\beta_z*O(WK_Z/Z) = O(WDB/Z) - \alpha_z*O(WP_Z/Z)$.

<mark>Equation 15)=18)+19) is fulfilled.</mark>

Decomposition of the changes in the deviations from the national average GVA per employed person

20) Change in the deviation from the national average GVA per employed person: $\Delta\,O(WDB/Z) = O(WDB/Z) - O(WDB/Z)_{(-1)}$ [percentage points].

21) Change in the deviation from the national average labour remuneration per employed person: $\Delta\,O(WP_Z/Z) = O(WP_Z/Z) - O(WP_{Z(-1)}/Z_{(-1)})$ [percentage points].

22) Change in the deviation from the national average capital remuneration per employed person: $\Delta\,O(WK_Z/Z) = O(WK_Z/Z) - O(WK_{Z(-1)}/Z_{(-1)})$ [percentage points].

17) Deviation from the national average capital remuneration per working person: $O(WK_P/P) = (WK_{P,i}/P_i - WK_P/P)/(WK_P/P)$ [percentages].

18) Contribution of labour remuneration in the deviation of the GVA from the national average per working person: $\alpha_p*O(WP_P/P) = O(WP_P/P)*UWP_P$ [percentage points]; α_p is not calculated intertemporally.

19) Contribution of labour remuneration in the deviation of the GVA from the national average per working person, theoretically: $\beta_p*O(WK_P/P) = O(WK_P/P)*(1-UWP_P)$ [percentage points]; β_p is not calculated intertemporally; in practice, calculated residually based on the assumption that: $\beta_p*O(WK_P/P) = O(WDB/P) - \alpha_p*O(WP_P/P)$.

<mark>Equation 15)=18)+19) is fulfilled.</mark>

Decomposition of the changes in the deviations from the national average GVA per working person

20) Change in the deviation from the national average GVA per working person: $\Delta\,O(WDB/P) = O(WDB/P) - O(WDB/P)_{(-1)}$ [percentage points].

21) Change in the deviation from the national average labour remuneration per working person: $\Delta\,O(WP_P/P) = O(WP_P/P) - O(WP_{P(-1)}/P_{(-1)})$ [percentage points].

22) Change in the deviation from the national average capital remuneration per working person: $\Delta\,O(WK_P/P) = O(WK_P/P) - O(WK_{P(-1)}/P_{(-1)})$ [percentage points].

23) Contribution of the labour remuneration in the change in the deviation from the national average GVA per employed person: $\alpha_z * \Delta O(WP_Z/Z) = \Delta O(WP_Z/Z)*(UWP_Z+UWP_{Z(-1)})/2$ [percentage points].	23) Contribution of the labour remuneration in the change in the deviation from the national average GVA per working person: $\alpha_p * \Delta O(WP_P/P) = \Delta O(WP_P/P)*(UWP_P+UWP_{P(-1)})/2$ [percentage points].
24) Contribution of the capital remuneration in the change in the deviation from the national average GVA per employed person, theoretically: $\beta_z * \Delta O(WK_Z/Z) = \Delta O(WK_Z/Z)*[(1-UWP_Z)+(1-UWP_{Z(-1)})]/2$ [percentage points], in practice calculated residually based on the assumption that: $\beta_z * \Delta O(WK_Z/Z) = \Delta O(WDB/Z) - \alpha_z * \Delta O(WP_Z/Z)$. <mark>Equation 20)=23)+24) is fulfilled</mark>	24) Contribution of the capital remuneration in the change in the deviation from the national average GVA per working person, theoretically: $\beta_p * \Delta O(WK_P/P) = \Delta O(WK_P/P)*[(1-UWP_P)+(1-UWP_{P(-1)})]/2$ [percentage points], in practice calculated residually based on the assumption that: $\beta_p * \Delta O(WK_P/P) = \Delta O(WDB/P) - \alpha_p * \Delta O(WP_P/P)$. <mark>Equation 20)=23)+24) is fulfilled</mark>

Accounting for factor resources (with TFP extraction)

Excel file: *II - Decomposition of the GVA growth rate into factor resources and TFP contributions for employed persons (Z)*	Excel file: *III - Decomposition of the GVA growth rate into factor resources and TFP contributions for working persons (P)*
<u>Decomposition of the real GVA growth rate into contributions of factor resources and TFP</u> 1) Real GVA growth rate: $\delta wdb = \Delta WDB/WDB_{(-1)}$ [percentages], where $\Delta WDB = WDB - WDB_{(-1)}$; **WDB** is in constant prices, $WDB_{(-1)}$ is in current prices. 2) Relative growth in the number of employed persons: $\delta z = \Delta Z/Z_{(-1)}$ [percentages], where $\Delta Z = Z - Z_{(-1)}$. 3) Real growth in capital value : $\delta k = \Delta K/K_{(-1)}$ [percentages], where $\Delta K = K - K_{(-1)}$, **K** is in constant prices, $K_{(-1)}$ is in current prices. 4) Share of labour remuneration (UWP_Z) in the GVA (WDB): $UWP_Z = WP_Z/WDB$ [percentages].	<u>Decomposition of the real GVA growth rate into contributions of factor resources and TFP</u> 1) Real GVA growth rate: $\delta wdb = \Delta WDB/WDB_{(-1)}$ [percentages], where $\Delta WDB = WDB - WDB_{(-1)}$; **WDB** is in constant prices, $WDB_{(-1)}$ is in current prices; 2) Relative growth in the number of working persons: $\delta p = \Delta P/P_{(-1)}$ [percentages], where $\Delta P = P - P_{(-1)}$. 3) Real growth in capital value: $\delta k = \Delta K/K_{(-1)}$ [percentages], where $\Delta K = K - K_{(-1)}$, **K** is in constant prices, $K_{(-1)}$ is in current prices. 4) Share of labour remuneration (UWP_P) in the GVA (WDB): $UWP_P = WP_P/WDB$ [percentages].

5) Labour contribution in the GVA growth: $\alpha_z*\delta z = \delta z*(UWP_Z+UWP_{Z(-1)})/2$ [percentage points].

6) Capital contribution in the GVA growth: $\beta_z*\delta k = \delta k*[(1-UWP_Z)+(1-UWP_{Z(-1)})]/2$ [percentage points]; it is assumed that: $\beta_z=1-\alpha_z$.

7) Contribution of total factor productivity (TFP_Z) in the GVA growth: theoretically, $\delta tfp_z = \Delta TFP_Z/TFP_{Z(-1)}$ [percentage points], calculated only residually: $\delta tfp_z = \delta wdb - \alpha_z*\delta z - \beta_z*\delta k$.

<mark>Equation 1)=5)+6)+7) is fulfilled.</mark>

Decomposition of the real GVA growth rate per employed person into contributions of capital and TFP

8) Real GVA growth rate per employed person:
$\delta wdb/z = \Delta(WDB/Z)/(WDB_{(-1)}/Z_{(-1)})$ [percentages],
$\Delta(WDB/Z) = WDB/Z - WDB_{(-1)}/Z_{(-1)}$; WDB is in constant prices, $WDB_{(-1)}$ is in current prices.

9) Real capital value growth rate per employed person: $\delta k/z = \Delta(K/Z)/(K_{(-1)}/Z_{(-1)})$ [percentages], $\Delta(K/Z) = K/Z - K_{(-1)}/Z_{(-1)}$; K is in constant prices, $K_{(-1)}$ is in current prices.

10) Contribution of the capital stock in GVA growth per employed person:
$\beta_z*\delta k/z = (\delta k/z)*[(1-UWP_Z)+(1-UWP_{Z(-1)})]/2$ [percentage points], it is assumed that: $\beta_z=1-\alpha_z$.

11) Contribution of the total factor productivity (TFP_Z) in the GVA growth per employed person, theoretically $\delta tfp_z/z = \Delta(TFP_Z/Z)/(TFP_{Z(-1)}/Z_{(-1)})$

5) Labour contribution in the GVA growth: $\alpha_p*\delta p = \delta p*(UWP_P+UWP_{P(-1)})/2$ [percentage points].

6) Capital contribution in the GVA growth: $\beta_p*\delta k = \delta k*[(1-UWP_P)+(1-UWP_{P(-1)})]/2$ [percentage points]; it is assumed that: $\beta_p=1-\alpha_p$.

7) Contribution of total factor productivity (TFP_P) in the GVA growth: theoretically, $\delta tfp_p = \Delta TFP_P/TFP_{P(-1)}$ [percentage points], calculated only residually: $\delta tfp_P = \delta wdb - \alpha_p*\delta p - \beta_p*\delta k$.

<mark>Equation 1)=5)+6)+7) is fulfilled.</mark>

Decomposition of the real GVA growth rate per working person into contributions of capital and TFP

8) Real GVA growth rate per working person:
$\delta wdb/p = \Delta(WDB/P)/(WDB_{(-1)}/P_{(-1)})$ [percentages],
$\Delta(WDB/P) = WDB/P - WDB_{(-1)}/P_{(-1)}$; WDB is in constant prices, $WDB_{(-1)}$ is in current prices.

9) Real capital value growth rate per working person: $\delta k/p = \Delta(K/P)/(K_{(-1)}/P_{(-1)})$ [percentages], $\Delta(K/P) = K/P - P_{(-1)}/P_{(-1)}$; K is in constant prices, $K_{(-1)}$ is in current prices;

10) Contribution of capital stock in GVA growth per working person:
$\beta_P*\delta k/p = (\delta k/p)*[(1-UWP_P)+(1-UWP_{P(-1)})]/2$ [percentage points], it is assumed that: $\beta_p=1-\alpha_p$.

11) Contribution of the total factor productivity (TFP_P) in the GVA growth per working person, theoretically $\delta tfp_p/p = \Delta(TFP_P/P)/(TFP_{P(-1)}/P_{(-1)})$

[percentage points], calculated only residually: $\delta tfp_Z/z$

$= \delta wdb/z - \beta_Z \cdot \delta k/z$.

Decomposition of the deviations from the national average GVA per employed person into contributions of capital and TFP

12) Deviation (**O**) from the national average GVA per employed person:
O(WDB/Z) = (WDB$_i$/Z$_i$ – WDB/Z)/(WDB/Z) [percentages], where **i** is the variable for a given voivodship, for a given section or for a given voivodship and section.

13) Deviation from the national average of the capital stock per employed person: **O(K/Z) = (K$_i$/Z$_i$ – K/Z)/(K/Z)** [percentages].

14) Contribution of the capital stock in the deviation of the GVA from the national average per employed person: **β_Z·O(K/Z) = O(K/Z)·(1-UWP$_Z$)** [percentage points]; it is assumed that: **β_Z=1- α_Z**; **α_Z** is not calculated intertemporally.

15) Contribution of the total factor productivity (TFP$_Z$) in the deviation of the GVA from the national average per employed person, theoretically:
O(TFP$_Z$/Z) = (TFP$_{Z,i}$/Z$_i$ – TFP$_Z$/Z)/(TFP$_Z$/Z) [percentage points],
O(TFP$_Z$/Z) = O(WDB/Z) – β_Z·O(K/Z) is calculated only residually.

Decomposition of the changes in the deviations from the national average GVA per employed person into contributions of capital and TFP

[percentage points], calculated only residually: $\delta tfp_P/p$

$= \delta wdb/p - \beta_P \cdot \delta k/p$.

Decomposition of the deviations from the national average GVA per working person into contributions of capital and TFP

12) Deviation (**O**) from the national average GVA per working person:
O(WDB/P) = (WDB$_i$/P$_i$ – WDB/P)/(WDB/P) [percentages], where **i** is the variable for a given voivodship, for a given section or for a given voivodship and section.

13) Deviation from the national average of the capital stock per working person: **O(K/P) = (K$_i$/P$_i$ – K/P)/(K/P)** [percentages].

14) Contribution of the capital stock in the deviation of the GVA from the national average per working person: **β_P·O(K/P) = O(K/P)·(1-UWP$_P$)** [percentage points]; it is assumed that: **β_P=1- α_P**; **α_P** is not calculated intertemporally.

15) Contribution of the total factor productivity (TFP$_P$) in the deviation of the GVA from the national average per working person, theoretically
O(TFP$_P$/P) = (TFP$_{P,i}$/P$_i$ – TFP$_P$/P)/(TFP$_P$/P) [percentage points],
O(TFP$_P$/P) = O(WDB/P) – β_P·O(K/P) is calculated only residually.

Decomposition of the changes in the deviations from the national average GVA per working person into contributions of capital and TFP

16) Change in the deviation from the national average GVA per employed person: $\Delta\,O(WDB/Z) = O(WDB/Z) - O(WDB/Z)_{(-1)}$ [percentage points].

17) Change in the deviation from the national average capital stock per employed person: $\Delta\,O(K/Z) = O(K/Z) - O(K_{(-1)}/Z_{(-1)})$ [percentage points].

18) Contribution of the capital stock in the change in the deviation of the GVA from the national average per employed person, theoretically $\beta_z * \Delta O(K/Z) = \Delta O(K/Z)*[(1-UWP_Z)+(1-UWP_{Z(-1)})]/2$ [percentage points]; it is assumed that $\beta_z = 1 - \alpha_z$; α_z is not calculated intertemporally;

19) Contribution of the total factor productivity (TFP$_Z$) in the change in the deviation of the GVA from the national average per employed person, theoretically $\Delta O(TFP_Z/Z) = \Delta(TFP_{Z,i}/Z_i - TFP_Z/Z)/(TFP_Z/Z)$ [percentage points], calculated only residually: ΔO $(TFP_Z/Z) = \Delta O(WDB/Z) - \beta_z * \Delta O(K/Z)$; it is assumed that: $\beta_z = 1 - \alpha_z$; α_z is not calculated intertemporally;

==Equation 16)=18)+19) is fulfilled==

16) Change in the deviation from the national average GVA per working person: $\Delta\,O(WDB/P) = O(WDB/P) - O(WDB/P)_{(-1)}$ [percentage points].

17) Change in the deviation from the national average capital stock per working person: $\Delta\,O(K/P) = O(K/P) - O(K_{(-1)}/P_{(-1)})$ [percentage points].

18) Contribution of the capital stock in the change in the deviation of the GVA from the national average per working person, theoretically $\beta_p * \Delta O(K/P) = \Delta O(K/P)*[(1-UWP_P)+(1-UWP_{P(-1)})]/2$ [percentage points]; it is assumed that $\beta_p = 1 - \alpha_p$; α_p is not calculated intertemporally;

19) Contribution of the total factor productivity (TFP$_Z$) in the change in the deviation of the GVA from the national average per working person, theoretically $\Delta O(TFP_P/P) = \Delta(TFP_{P,i}/P_i - TFP_P/P)/(TFP_P/P)$ [percentage points], calculated only residually: ΔO $(TFP_P/P) = \Delta O(WDB/P) - \beta_p * \Delta O(K/P)$; it is assumed that: $\beta_p = 1 - \alpha_p$; α_p is not calculated intertemporally;

==Equation 16)=18)+19) is fulfilled==

Note. The periods are understood as yearly, i.e. subscript (-1) indicates the values for the previous year and values without subscripts refer to the current period. All values should be additionally indexed by sections, voivodships or sections and voivodships at the same time (possibly other selected aggregations) when they concern them. Gross value added (GVA) and capital (K) are not indexed, as they are the same for employed and working persons. The explanation of the symbols is provided in Table 2.

Table 2. Annotated explanation of the symbols used in Table 1

Symbol	Variable	Unit	Type of data	Notes
WDB	Gross value added (GVA)	Monetary units (e.g. PLN)	Input data from the Local Data Bank (BDL) of Statistics Poland,	In terms of the project requirements, data by voivodships and sections were available only at current prices; data at constant prices were estimated by means of transmission tables to

			computed in constant prices	Eurostat, broken down by divisions and sections
WP_Z WP_P	Labour remuneration	Monetary units (e.g. PLN)	Input data from BDL of Statistics Poland, computed in constant prices and for working persons	Within the scope of the project, data by voivodships and sections were available only in current prices for employed persons (Z); data in constant prices and for working persons (P) were estimated by means of transmission tables to Eurostat
UWP_Z UWP_P	Share of labour remuneration in GVA	Percentages	Computed result data	Used to calculate elasticities α_z and α_p
ΔUWP_Z ΔUWP_P	Growth in the share of the labour remuneration in GVA	Percentage points	Computed result data	Optional additional information
RK_Z RK_P	Gross capital profitability	Percentages	Computed result data	Ratio of income of capital owners (including the income from the self-employment - for employed persons Z) or of income from capital (without the income from the self-employment - for working persons P) to the value of capital
WK_Z WK_P	Capital remuneration	Percentages	Computed	Income of capital owners (including the income with from the self-employment (- for employed persons – Z) or income from capital (without the income from the self-employment - for working persons P)
ΔRK_Z ΔRK_P	Change in the gross capital profitability	Percentage points	Computed result data	Applies to employed persons Z or working persons P per item RK_Z and RK_P
δwdb	Real GVA growth rate (relative)	Percentages	Computed result data	Ratio of ΔWDB to GVA in current prices for the previous year; price comments as under item WDB
ΔWDB	Real GVA growth rate (absolute)	Monetary units(e.g. PLN)	Computed	Difference between GVA for the current year at constant prices and GVA for the previous year at current prices
δwp_z δwp_p	Real labour remuneration growth rate (relative)	Percentages	Computed result data	Ratio of ΔWP to WP in current prices of the previous year; price comments as under items WP_Z and WP_P

ΔWP_Z ΔWP_P	Real labour remuneration growth rate (absolute)	Monetary units (e.g. PLN)	Computed	Difference between the current year's labour remuneration at constant prices and the previous year's labour remuneration at current prices
Δwk_z Δwk_p	Real capital remuneration growth rate (relative)	Percentages	Computed result data	Ratio of ΔWK to WK for the previous year; comment as under items WK_Z and WK_P
ΔWK_Z ΔWK_P	Real labour remuneration growth rate (absolute)	Monetary units (e.g. PLN)	Computed	Difference between the current year's capital remuneration at constant prices and the previous year's capital remuneration at current prices
$\alpha_z*\delta wp_z$ $\alpha_p*\delta wp_p$	Contribution of labour remuneration in GVA growth	Percentage points	Computed result data	Real relative labour remuneration growth (δwp) weighted by share of labour in the economy α for a given aggregation; data for employed persons (Z) differ from those for working persons (P)
$\beta_z*\delta wk_z$ $\beta_p*\delta wk_p$	Contribution of labour remuneration in GVA growth	Percentage points	Theoretical value	In practice calculated residually as the difference between δwdb and $\alpha*\delta wp$ with appropriate Z or P indices
$\delta wdb/z$ $\delta wdb/p$	Real GVA growth rate per employed or working person (relative)	Percentages	Computed result data	GVA and the number of employed persons Z or working persons P change over time; price comments as under item **WDB**
$\delta wp_z/z$ $\delta wp_p/p$	Real labour remuneration growth rate per employed or working person (relative)	Percentages	Computed result data	Labour remuneration and the number of employed persons Z or working persons P change over time; price comments as under items WP_Z and WP_P
$\delta wk_z/z$ $\delta wk_p/p$	Real capital remuneration growth rate per employed or working person (relative)	Percentages	Computed result data	Capital remuneration and the number of employed persons Z or working persons P change over time; price comments as under items WK_Z and WK_P
$\alpha_z*\delta wp_z/z$ $\alpha_p*\delta wp_p/p$	Contribution of labour remuneration in GVA growth per employed or working person (relative)	Percentage points	Computed result data	Labour remuneration, GVA and the number of employed persons Z or working persons P change over time; price comments as under items WP_Z and WP_P
$\beta_z*\delta wk_z/z$ $\beta_p*\delta wk_p/p$	Contribution of capital remuneration in GVA growth per employed or working person (relative)	Percentage points	Theoretical value	In practice calculated residually as the difference between $\delta wdb/z$ and $\alpha*\delta wp/z$ with appropriate Z or P indices

O(WDB/Z) O(WDB/P)	Deviation from the national average GVA per employed or working person	Percentages	Computed result data	GVA and the number of employed persons Z or working persons P change in space; price comments as under item **WDB**
O(WP$_Z$/Z) O(WP$_P$/P)	Deviation from the national average labour remuneration per employed or working person	Percentages	Computed result data	Labour remuneration, GVA and the number of employed persons Z or working persons P change in space; price comments as under items **WP$_Z$** and **WP$_P$**
O(WK$_Z$/Z) O(WK$_P$/P)	Deviation from the national average capital remuneration per employed or working person	Percentages	Computed result data	Capital remuneration, GVA and the number of employed persons Z or working persons P change in space; price comment as under items **WK$_Z$** and **WK$_P$**
α$_z$*O(WP$_Z$/Z) α$_p$*O(WP$_P$/P)	Contribution of labour remuneration in the deviation from the national average GVA per employed or working person	Percentage points	Computed result data	Capital remuneration, GVA and the number of employed persons Z or working persons P change in space; **α** (with appropriate indices) is not computed intertemporally
β$_z$*O(WK$_Z$/Z) β$_p$*O(WK$_P$/P)	Contribution of capital remuneration in the deviation from the national average GVA per employed or working person	Percentage	Theoretical value	In practice residually calculated result data as the difference between **O(WDB/Z)** and **α*O(WP/Z)** with appropriate indices
ΔO(WDB/Z) ΔO(WDB/P)	Change in the deviation from the national average GVA per employed or working person	Percentage points	Computed result data	GVA and the number of employed persons Z or working persons P change over time and space
ΔO(WP$_Z$/Z) ΔO(WP$_P$/P)	Change in the deviation from the national average labour remuneration per employed or working person	Percentage points	Computed result data	GVA and the number of employed persons Z or working persons P change over time and space
ΔO(WK$_Z$/Z) ΔO(WK$_P$/P)	Change in the deviation from the national average capital remuneration per employed or working person	Percentage points	Computed result data	GVA and the number of employed persons Z or working persons P change over time and space
α$_z$*O(WP$_Z$/Z) α$_p$*O(WP$_P$/P)	Contribution of labour remuneration in the change in the deviation from the national average GVA per employed or working person	Percentage points	Computed result data	GVA and the number of employed persons Z or working persons P change over time and space

$\beta_z \cdot O(WK_Z/Z)$ $\beta_p \cdot O(WK_P/P)$	Contribution of capital remuneration in the change in the deviation from the national average GVA per employed or working person	Percentage points	Theoretical value	In practice residually calculated result data as the difference between $\Delta O(WDB/Z)$ and $\alpha \cdot \Delta O(WP/Z)$ with appropriate indices
Z	Number of employed persons	Persons	Input data from BDL	Data available by voivodships and by sections or groups of sections to the extent required by the project
P	Number of working persons	Persons	Computed	Data unavailable in BDL of Statistics Poland in the scope required by the project, calculated by means of transmission tables to Eurostat by sections and divisions
ΔZ ΔP	Growth in the number of employed or working persons (absolute)	Persons	Computed	–
δz δp	Growth in the number of employed or working persons (relative)	Percentages	Computed	–
$\alpha_z \cdot \delta z$ $\alpha_p \cdot \delta p$	Labour contribution (employed persons Z or working persons P) in GVA growth	Percentage points	Computed result data	Elasticity α with appropriate indices is common for both types of decompositions, i.e. with and without a TFP extraction
K	Capital stock	Monetary units (e.g. PLN)	Input data from BDL in historical prices; in constant and current prices – computed	In the scope of the project, data by voivodships and by sections are available only at historical prices; at current and constant prices the data are estimated by means of transmission tables to Eurostat broken down by divisions and sections
ΔK	Growth in capital stock	Monetary units (e.g. PLN)	Computed	Comments as under item K
$\beta_z \cdot \delta k$ $\beta_p \cdot \delta k$	Capital contribution (employed persons Z or working persons P) in GVA growth	Percentage points	Computed result data	In this case, elasticity β is calculated as $\beta = 1 - \alpha$, with appropriate indices

δtfp_z δtfp_p	TFP contribution (employed persons Z or working persons P) in GVA growth	Percentage points	Computed result data	Result data calculated only residually as the difference between **δwdb** and **αz*δz** and **βz*δk** for employed persons or between **δwdb** and **αp*δp** and **βp*δk** for working persons
$\delta k/z$ $\delta k/p$	Real growth in capital stock per employed or working person (relative)	Percentages	Computed result data	Capital K and the number of employed persons Z or working persons P change over time; comments as under item **K**
$\beta_z*\delta k/z$ $\beta_p*\delta k/p$	Capital contribution in GVA growth per employed persons Z or working persons P	Percentage points	Computed result data	In this case elasticity **β** is calculated as **β =1 – α**, with appropriate indices
$\delta tfp_z/z$ $\delta tfp_p/p$	TFP contribution in GVA growth per employed persons Z or working persons P	Percentage points	Computed result data	Result data calculated only residually as the difference between **δwdb/z** and **αz*δz/z** and **βz*δk/z** for employed persons or between **δwdb/p** and **αp*δp/p** and **βp*δk/p** for working persons
O(K/Z) **O(K/P)**	Deviation in the capital value per employed persons Z or working persons P	Percentages	Computed	Comments as under items **Z**, **P** and **K**
βz*O(K/Z) **βp*O(K/P)**	Contribution of the deviation in the capital value per employed persons Z or working persons P from the national average to the deviation of GVA per employed persons Z or working persons P	Percentage points	Computed result data	Comments as under items **Z**, **P** and **K**; in this case elasticity **β** is calculated as **β =1 – α**, with appropriate indices
O(TFPZ/Z) **O(TFPP/P)**	Contribution of the deviation in TFP per employed persons Z or working persons P from the national average to the deviation of GVA per employed persons Z or working persons P	Percentage points	Computed result data	Result data calculated only residually as the difference between **O(WDB/Z)** or **O(WDB/P)** and **βz*O(K/Z)** or **βp*O(K/P)**

Δ O(K/Z) **ΔO(K/P)**	Change in the deviation of the capital value per employed persons Z or working persons P	Percentages	Computed	Comments as under items **Z**, **P** and **K**
β$_z$*ΔO(K/Z) **β$_p$*ΔO(K/P)**	Contribution of the change in the deviation of the capital value per employed persons Z or working persons P from the national average to the deviation of GVA per employed persons Z or working persons P	Percentage points	Computed result data	Comments as under items **Z**, **P** and **K**; in this case elasticity **β** is calculated as **β =1 – α**, with appropriate indices
Δ O(TFP$_Z$/Z) **Δ O(TFP$_P$/P)**	Contribution of the change in the deviation of TFP per employed persons Z or working persons P from the national average to the deviation of GVA per employed persons Z or working persons P	Percentage points	Computed result data	Result data calculated only residually as the difference between **Δ O(WDB/Z)** or **Δ O(WDB/P)** and **β$_z$*ΔO(K/Z)** or **β$_p$*ΔO(K/P)**

Note. Index **Z** is the symbol for employed persons and index **P** is the symbol for working persons. GVA and capital are not indexed, as they are the same for employed and working persons. **Δ** denotes absolute growth, **δ** – relative growth (**δx = Δx/x**).