

**Methodology
of gross value added growth decomposition
in KLEMS productivity accounts
for the Polish economy**

by Dariusz Kotlewski, PhD
in co-operation with:
Mirosław Błażej

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Introduction

The name of KLEMS economic productivity accounts originates from symbols traditionally used in formal formulae or from the first letters in English language (K – Capital, L – Labour, E – Energy, M – Materials, S – Services). It indicates the factors that are referred to in the accounts. These are the so-called “primary factors”, i.e. “capital” and labour”, and “secondary factors”, which are the components of “intermediate consumption”, i.e. “energy”, “materials” that include both raw materials and semi-finished products and “services”, being the contributions to firms from outside. The KLEMS productivity accounts present economic processes *ex post*, from the supply side point of view. They evolved from a neoclassical economic growth theory formulation in the form of Solow’s decomposition from the 1950s. This theory is basic for two main methodology options presently carried out worldwide. One of them is the OECD methodology, aiming more at maximising international comparability, even at the expense of some compromises against the theory. The second is the KLEMS methodology, being more *de rigueur* with the mainstream theory that is fundamental for this accounting. In the EU KLEMS consortium participate a number of European countries (10 in 2016) and more of them are supposed to be participating afterward (some more countries are presented already, but with very short time series or no growth decompositions most essential in the KLEMS productivity accounts). It has a variation in the form of WORLD KLEMS that is intended to become a global platform, whereas EU KLEMS is basically a regional initiative. The present coverage concerns KLEMS economic productivity accounts elaborated methodologically and calculated for Poland in the Department of Macroeconomic Studies and Finance of Statistics Poland and is based on data delivered by the National Accounts Department and Demographic Surveys and Labour Market Department of Statistics Poland.

1. Definition of economic *multifactor productivity* (MFP)

In the KLEMS economic productivity accounts the following formula is used for gross output growth at a given aggregation level j for period t (Timmer et al. 2007a)¹:

$$\Delta \ln Y_{jt} = \bar{v}_{jt}^X \Delta \ln X_{jt} + \bar{v}_{jt}^K \Delta \ln K_{jt} + \bar{v}_{jt}^L \Delta \ln L_{jt} + \Delta \ln A_{jt}^Y \quad (1)$$

where Y is gross output, X – intermediate consumption, K – capital, L – labour, whereas A^Y is the so-called *multifactor productivity* (MFP), that can be considered as a variant of TFP. These values are subscribed to indicate that they concern industries j and periods t . Δ , for all values under this symbol, denotes changes between periods t and $t - 1$, usually considered as yearly periods. If the changes are small, as it is usually the case for yearly periods, we can approximate that $\Delta \ln x = \Delta x / x$, which allows to interpret these changes as relative changes, e.g. expressed in percentages. \bar{v} with appropriate superscripts and subscripts denote average value shares of the given factors income (indicated in superscripts as X , K or L) in the gross output, between periods t and $t - 1$, that are calculated according to formula $\bar{v} = (v_t + v_{(t-1)})/2$ (for simplicity the subscript j present in formula (1) has been omitted here).

The shape of formula (1) results from bestowing to the production function a trans-logarithmic formulation, in order to bring to it an additive property for the relative changes of its independent variables. It is of basic importance, as it eases the use of statistical methods. The growth of A^Y value, i.e. MFP, is calculated as a residual, therefore the formula (1) is always met.

It is quite common in statistics that intermediate consumption deflators, necessary for computing real values, are missing. Moreover, because of large differences between the countries as far as vertical integration of firms is considered, the share of intermediate consumption in gross output is very different across different countries. Therefore, for international comparisons it is justified to use a decomposition of gross value added growth instead². So, on the EU KLEMS platform it is the gross value added growth decomposition which is carried out extensively. A similar trans-log function is used here, but without the intermediate consumption X as a component "factor":

$$\Delta \ln V_{jt} = \bar{w}_{jt}^K \Delta \ln K_{jt} + \bar{w}_{jt}^L \Delta \ln L_{jt} + \Delta \ln A_{jt}^V \quad (2)$$

where V stands for gross value added, and the other symbols (with appropriate superscripts and subscripts) have the same meaning as in formula (1) but, with the exception of capital K and labour L , take different values. It is important to note that the analogical average shares \bar{w} (of production factors remuneration in the gross value added) are not identical to average shares \bar{v} from formula (1) (they are

¹ KLEMS productivity accounts are based mainly on the works of Dale Jorgenson and associates (1963, 1967, 1987, 1989, 2005).

² Lastly, however, the gross output growth decomposition has been performed in Statistics Poland, which has an additional analytical merit.

expressed in percentages and calculated in a similar way to average shares \bar{v}). Also, the contribution of MFP in the decomposition of gross value added V growth is not identical in percentages to the contribution of MFP in the decomposition of gross output Y growth, although its absolute growth in the ideal case where there were no variability associated with intermediate consumption should be identical.

2. Gross output, intermediate consumption and gross value added accounts

The gross output growth in industry j in period t , i.e. exactly the left hand side of the equation (1), is defined as the sum of weighted contributions of the growths of individual products i to the overall growth of the entire industry j in value terms:

$$\Delta \ln Y_{jt} = \sum_i \bar{v}_{ijt}^Y \Delta \ln Y_{it} \quad (3)$$

where \bar{v} with appropriate superscript and subscripts stands for average value shares of the individual products i of the given industry j in gross output Y of the entire industry j , between the periods t and $t - 1$ (they are calculated similarly, according to formula $\bar{v} = (v_t + v_{(t-1)})/2$).

The growth of intermediate consumption in the given industry can be defined by the formula:

$$\Delta \ln X_{jt} = \sum_i \bar{v}_{ijt}^X \Delta \ln X_{it} \quad (4)$$

where subscript i stands for individual products as above, whereas \bar{v} with the appropriate superscript and subscripts stands for average value shares of the individual products i of the given industry j in intermediate consumption X of industry j , between the periods t and $t - 1$, calculated in a similar way to the shares in formula (3).

Similarly, the growth of gross value added can be defined by the formula:

$$\Delta \ln V_{jt} = \sum_i \bar{w}_{ijt}^V \Delta \ln V_{it} \quad (5)$$

where V stands for gross value added, and the superscript V for w indicates that the shares are in the gross value added.

To calculate both the gross output and the intermediate consumption supply and use tables (SUT) can be used as second best, instead of symmetric input-output tables (IOT), which are available only for some years – on the condition, however, that they are consistent with SNA³ and ESA⁴ systems and their updates. The Eurostat transmission tables are consistent with ESA, and the above-mentioned SUT tables. Therefore, they can also be used in KLEMS accounting as data source.

³ System of National Accounts.

⁴ European System of Accounts.

3. Labour factor accounts

Similarly to the growths of gross output, intermediate consumption and gross value added can be defined the growth of labour factor at industry j level, according to the formula:

$$\Delta \ln L_{jt} = \sum_l \bar{v}_{ljt} \Delta \ln H_{ljt} \quad (6)$$

where L denotes the labour factor (understood as labour services), l – the different kinds of labour, v with appropriate subscripts – average income value shares of the different labour kinds l from two periods t and $t - 1$ (calculated in a similar way to previously mentioned shares as arithmetic averages), and H with appropriate subscripts – the number of hours worked in the given labour kind l in period t . The labour kinds are distinguished according to sexes, three age groups and three education attainment levels. It is assumed here that the values of labour services of all kinds l are proportional to the hours worked in these labour kinds and that the employees of each labour kind are remunerated according to their marginal productivity, which translates into their compensation. Hence, we have the value shares of labour kinds, as their compensation shares in the overall compensation of industry j . The labour-kind shares \bar{v}_{ljt} in industries j are calculated similarly to products i shares in industries j in previous formulae (3), (4) and (5). Thus we have 18 kinds of labour l , i.e. the formula (6) can be specified as:

$$\Delta \ln L_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln H_{ljt} \quad (7)$$

The labour factor understood in this way (as labour services) can be decomposed into one component consisting of sheer growth of the number of hours worked, and the other component related to labour quality understood as labour composition LC change, and the appropriate formula for the growth of this labour quality is:

$$\Delta \ln LC_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln H_{ljt} - \Delta \ln H_{jt} \quad (8)$$

However, the labour factor can also be extended by remuneration growth effect. Then the formula for labour quality SC understood in this way is:

$$\Delta \ln SC_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln W_{ljt} - \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln H_{ljt} \quad (9)$$

where W are compensations of the given labour kinds l , in the given industry j , in period t , which should be consistent with:

$$\Delta \ln SC_{jt} = \Delta \ln W_{jt} - \Delta \ln L_{jt} \quad (10)$$

basing on formula (6) and the observation that the first term on the right hand side of equation (9) is nothing else than the sum of weighted compensation growths.

Labour quality on the EU KLEMS platform (also on the World KLEMS one) is understood as labour composition, but in Statistics Poland the calculations have been done in both ways. When addressing these theoretical issues, the problem is that often the labour factor is measured in inappropriate units, i.e. employees instead of hours, which are often paid hours instead of worked hours. The phenomenon of self-employment is also an issue to be addressed. Some data adjustments were therefore necessary. The values of the labour factor were adjusted for the self-employment, thanks to the assumption that hours worked of the self-employed are remunerated in the same way as hours worked of the contractually employed according to the NACE activities respectively. This way is generally practiced by all KLEMS performing countries.

4. Capital factor accounts

The growth of the capital factor can be similarly defined according to formula:

$$\Delta \ln K_{jt} = \sum_k \bar{v}_{kjt} \Delta \ln A_{kj} \quad (11)$$

In this formula the symbols K for the capital factor and A for the individual assets have replaced L and H used in the formula (6) for the labour factor. The average shares \bar{v} from two periods are calculated similarly for individual asset kinds k , i.e. as shares in the overall capital remuneration of industry j . Therefore, the growth of capital factor is defined in the KLEMS productivity accounts as the growth of capital services, not as the growth of capital assets⁵.

A division of capital assets into 9 kinds is required:

- 1) Dwellings
- 2) Other structures and buildings
- 3) Transport equipment
- 4) Other machinery and equipment
- 5) Computing equipment
- 6) Communication equipment
- 7) Cultivated assets
- 8) Intellectual property products
- 9) Software

In particular, three kinds of capital, i.e. 5) computing equipment, 6) communication equipment and 9) software are not being extracted from other aggregates in the Polish conditions, and therefore this additional operation had to be carried out. After this extraction and after the application of formula (11),

⁵ The arbitrage equation derived from the neo-classical theory of investment is used here, as introduced by Jorgenson (1963) and Jorgenson & Griliches (1967). It is assumed that the relative growths of "capital services" are equal to the relative growths of capital asset stocks (although their absolute growths are not) at the disaggregated level. From it arises the necessity to operate only with relative growths (expressed e.g. in percentages) in the entire KLEMS accounts.

these three categories are aggregated into the so called ICT capital, whereas the remaining capital items are joined in the so called non-ICT capital. In this way the capital factor is divided into two sub-factors:

$$\bar{w}_{jt}^K \Delta \ln K_{jt} = \bar{w}_{jt}^{KIT} \Delta \ln KIT_{jt} + \bar{w}_{jt}^{KNIT} \Delta \ln KNIT_{jt} \quad (12)$$

where *KIT* denotes the ICT capital and *KNIT* – the non-ICT capital. The extraction of the ICT capital into a separated sub-factor was based on the conviction that it is especially important for the KLEMS accounts, but this fact was confirmed only for some countries⁶.

There are controversies whether dwellings should be treated as production capital in this accounts. Because of Polish dwelling market specificities, at first it has been decided not to consider the residential capital. But from the point of view of international comparisons it is needed, because the accounts for the above-mentioned European countries belonging to the EU KLEMS system are carried out together with the dwellings. In Statistics Poland the calculations have been done for both of these assumptions and the results are published accordingly. Therefore, since we have two ways of calculating labour quality (*LC* or *SC* as above mentioned) and two ways of calculating capital values (with and without residential capital), the results are published for the four possible variants indicated as A, B, C and D, of which the B version is the closest to the EU KLEMS platform way of calculating and therefore it is the one that should be used in international comparisons.

The basic way to assess the stock of capital assets is to assume some initial capital stock in some base year and to calculate the changes by subtracting capital depreciation and adding up investments⁷ according to the so called *perpetual inventory method*. This kind of activity is carried out in the National Accounts, but on different NACE 2 aggregates than in the EU KLEMS accounts. Because all 34 EU KLEMS aggregations are at higher or at the same levels as NACE aggregations (divisions) it was possible to use the data from the National Accounts by summing them up to EU KLEMS aggregations. In the EU KLEMS methodology the capital depreciation is calculated according to the USA capital depreciation indices (geometrical), which can be used for all countries and are used by EU KLEMS participants. However, some of these values are provided as value ranges, so for individual countries it remains necessary to individually assess them, which is usually already done in the National Accounts.

There are some problems with international comparability of some kinds of capital. In different countries are used different definitions (narrow and wide) of IT and CT industries. The initial capital stocks are also an issue for international comparability, as they are differently assessed in different countries and for different base years. There is also the problem of leased capital services. The services of capital that is not owned by the user are usually considered as a kind of intermediate consumption, instead of being considered as a component of the capital factor (at the same time as above mentioned, the individual countries are different as far as vertical integration of firms is considered, therefore the

⁶ In the KLEMS accounts done in the USA it is not practiced. Rather the intensity of ICT use in different industries is being analysed.

⁷ In this elaboration we differentiate linear (arithmetic) depreciation provisions practiced by the accountants and the capital depreciation which in the KLEMS methodology is non-linear (geometric).

intermediate consumption is hidden within the vertically integrated firms to a different degree in different countries). Moreover, sometimes there is the issue of public participation in infrastructural investments, which should be treated in the same way as private investments. And in addition, the countries treat differently the price deflation which is present and specific for ICT products. Therefore, the price indices applied by different countries for the ICT industry are very different, and it is so even for countries with similar economies as in Europe.

5. Calculation methodology for Poland KLEMS

It seems justified to carry out KLEMS productivity accounting for Poland basing on the assumptions as close as possible to those applied by the EU KLEMS countries. The basing approach is to apply only a gross value added growth decomposition, which helps international comparisons, although lastly (from 2019) the decomposition of gross output growth has been carried out as well in Statistics Poland (published in parallel on the internet site).

For the labour factor, the data from a representative survey called Z-12 are available for the even years 2004, 2006, 2008, 2010, 2012, 2014 and 2016, etc. Therefore, they are sufficient to carry out the KLEMS productivity accounts for Poland flung down to 2005. For the uneven years linear interpolation has been applied. For 2004 these data concern the number of full-time employees, the average remunerations by hour worked in the nominal time and in the overtime of full-time employees in the entire year, and the number of hours worked of these full-time employees. From 2006 onward, the data concern all employees, not only full-time employees. Because these data are adjusted upward by a structure concerning the entire labour market, the possible errors here become negligible.

The data for 2004 and 2006 are in the NACE 1 classification system. From 2008 onward the data are in the NACE 2 classification system, but the data for 2008 have been converted also into NACE 1. In EU KLEMS a fixed correspondence between NACE 1 and NACE 2 is applied for the labour factor and the same has been practiced in the KLEMS accounts for Poland (subdivision into 14 sections and section groups). The labour factor has been assessed, considering the self-employment, for the total number of hours worked and total compensation of this factor⁸.

Then, for the capital factor the basic operation was to extract the above-mentioned three kinds of ICT capital before aggregating them into a single category of ICT capital. This was done thanks to supply and use tables (SUT), where there are figures in the column „gross fixed capital formation” for all three kinds of ICT capital. These three figures were distributed proportionally to the horizontal structure of software services taken from the same supply and use tables, which was transposed in a vertical position and aggregated into 34 vertical KLEMS aggregations, basing on the assumption that at the industry aggregation level the values of software services are quite proportional to the values of the three categories of ICT capital. The capital stocks calculated in this way were then extracted from aggregates where they were contained before. Because the supply and use tables are elaborated in two different

⁸ The work to convert all data into NACE 2 system is being carried out.

classification systems NACE 1 and NACE 2 and they will not be recalculated, it was necessary to use the above-mentioned correspondence of 14 sections and section groups applied for the labour factor. For the latest years in the performed KLEMS accounting the last available supply and use tables were applied, since they are done with a greater delay in comparison to other data sources. The practice to use supply and use tables from previous years is quite common since the structures of data in these tables change very slowly.

One expected problem in KLEMS productivity accounts was the passing from ESA 95 system to ESA 2010 system, because not all data were already recalculated into the new system, and some data shall never be recalculated (it concerns supply and use tables from before 2010). Therefore, there was sometimes the need to use mixed data (it is applied only for the ICT capital extraction). To test whether this is acceptable, an analysis was carried out for differences in places where the data are available in both systems, i.e. by comparing the differences of capital stock growths expressed in both classifications. The results of this analysis have shown that the inconsistencies are negligible from the point of view of KLEMS productivity accounting requirements.

One specific solution has been applied in the publication of result data for Poland KLEMS. Data are presented as contributions to aggregate level gross value added growth or contributions to industry level gross value added growths. The first of this way of presenting the data (data marked by A, B, C and D) has the merit of indicating the weight of the different industries in the aggregate gross value added growth and is a specific Poland KLEMS solution. The second way (marked by A', B', C' and D' on the same Excel sheets), practiced more universally, allows to compare different industries on the same graphs more easily.

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