

## Methodology

### of developed labour factor decomposition in KLEMS productivity accounts for the Polish economy

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A developed labour factor decomposition that includes a detailed industry level decomposition by labour kinds seems to open new avenues for economic analysis. This concerns in particular the changes in the business cycle, the labour market and economic shocks (both external and arising from country situation and its policy) resilience. It can also allow to build connections with other studies.

In KLEMS accounting the contribution of the relative growth of the labour factor is being divided into the contributions of two sub-factors:

$$\bar{w}_{jt}^L \Delta \ln L_{jt} = \bar{w}_{jt}^L \Delta \ln LC_{jt} + \bar{w}_{jt}^L \Delta \ln H_{jt} \quad (1)$$

where:

$$\Delta \ln LC_{jt} = \sum_l \bar{v}_{l,jt} \Delta \ln H_{l,jt} - \Delta \ln H_{jt} \quad (2)$$

In the above-mentioned formulae  $\bar{w}_{jt}^L$  is the average share of the labour factor in GVA of industry  $j$  for two discrete periods  $(t-1)$  and  $t$ ;  $\Delta \ln L_{jt}$  is the relative growth of the labour factor value in industry  $j$  between two discrete periods  $(t-1)$  and  $t$ ; and  $\Delta \ln H_{jt}$  is the relative growth of the number of hours worked in industry  $j$  between these two discrete periods.  $\Delta \ln LC_{jt}$  is the relative change in the so called labour composition (otherwise called labour quality) in industry  $j$  between two discrete periods  $(t-1)$  and  $t$ , understood as an effect of change in the structure of the labour factor from the point of view of different labour kinds' shares  $l$ , calculated residually by subtracting the contribution of hours worked in the given industry between these two discrete periods, i.e.  $\Delta \ln H_{jt}$ , from the sum of weighted contributions of different labour kinds, i.e.  $\bar{v}_{l,jt} \Delta \ln H_{l,jt}$ .  $\Delta \ln H_{l,jt}$  is the relative growth of the number of hours worked in industry  $j$  between two discrete periods  $(t-1)$  and  $t$  for different labour kinds  $l$ , whereas

$\bar{v}_{l,jt}$  are average shares of labour kinds  $l$  in labour compensation in industry  $j$  between two discrete periods  $(t-1)$  and  $t$ . In this way the traditionally understood (following R. Solow) contribution of the labour factor to GVA growth as the contribution of hours worked is complemented by the contribution of labour composition, that was not extracted before from the so called Solow residual. In KLEMS accounting 18 kinds of labour are defined which arises from division into sexes, three age groups and three education attainment levels.

This analysis of the labour factor can be furthered, however. The contribution of hours worked growth in formula (1) can be decomposed by changing this formula into:

$$\bar{w}_{jt}^L \Delta \ln L_{jt} = \bar{w}_{jt}^L \Delta \ln LC_{jt} + \bar{w}_{jt}^L \Delta \ln M_{jt} + \bar{w}_{jt}^L \Delta \ln H_{Mjt} \quad (3)$$

where:

$$\Delta \ln H_{Mjt} = \Delta \ln H_{jt} - \Delta \ln M_{jt} \quad (4)$$

In the above-mentioned formulae  $\Delta \ln H_{Mjt}$  is the relative growth of hours worked per employee in industry  $j$  between two discrete periods  $(t-1)$  and  $t$ , calculated residually by subtracting the relative growth of the number of employees, i.e.  $\Delta \ln M_{jt}$ , from the relative growth of hours worked, i.e.  $\Delta \ln H_{jt}$ . This technique of residual calculations is the reason why the formulae (1) and (3) are always met.

The analysis of the labour factor can also be extended. If the contribution of labour ( $L$ ) is subtracted from the contribution of labour compensation ( $LR$ ) then we receive the contribution of change in the relative level of remunerations ( $SC$ ) according to the following formula:

$$\bar{w}_{jt}^L \Delta \ln SC_{jt} = \bar{w}_{jt}^L \Delta \ln LR_{jt} - \bar{w}_{jt}^L \Delta \ln L_{jt} \quad (5)$$

In such a case the contributions of all the above-mentioned entities of the labour factor can be joined in the formula:

$$\bar{w}_{jt}^L \Delta \ln LR_{jt} = \bar{w}_{jt}^L \Delta \ln SC_{jt} + \bar{w}_{jt}^L \Delta \ln LC_{jt} + \bar{w}_{jt}^L \Delta \ln M_{jt} + \bar{w}_{jt}^L \Delta \ln H_{Mjt} \quad (6)$$

In the KLEMS accounting labour composition  $LC$  is interpreted as the main manifestation of labour efficiency in the long run<sup>1</sup>, which only to some degree translates into the actual remunerations' level. The remaining remunerations' level change  $SC$  can be attributed to the actual labour usage that mostly can be related with the business cycle (although, this residually calculated component of labour remuneration level change can also be impacted by all sorts of other 'factors' such as labour factor flows to activities with higher remunerations resulting from temporary vogues for some products,

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<sup>1</sup> The neoclassical premise is that labour is being remunerated according to its marginal productivity.

individual industry crises related to missing and expensive resources, interactions with the capital factor, i.e. the availability of the capital for labour against the availability of labour for the capital<sup>2</sup>, etc.).

For clarity, the Excel tables concerning this developed decomposition of the labour factor are presented in a hierarchical way following the equation (6) divided into three equations:

$$\begin{aligned}\bar{w}_{jt}^L \Delta \ln LR_{jt} &= \bar{w}_{jt}^L \Delta \ln SC_{jt} + \bar{w}_{jt}^L \Delta \ln L_{jt} \\ \bar{w}_{jt}^L \Delta \ln L_{jt} &= \bar{w}_{jt}^L \Delta \ln LC_{jt} + \bar{w}_{jt}^L \Delta \ln H_{jt} \\ \bar{w}_{jt}^L \Delta \ln H_{jt} &= \bar{w}_{jt}^L \Delta \ln M_{jt} + \bar{w}_{jt}^L \Delta \ln H_{Mjt}\end{aligned}\tag{7}$$

They represent the three stages of the labour factor decomposition shown on the three graphs of the last spreadsheet of the related Excel file.

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<sup>2</sup> It is about the income division between the labour and capital arising from their relative bargaining powers (that to some degree can arise from social regulations).