CHANNEL PERFORMANCE UNDER VENDOR MANAGED CONSIGNMENT INVENTORY CONTRACT WITH ADDITIVE STOCHASTIC DEMAND

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ABSTRACT

Consignment as the shifting of the inventory ownership to a supplier is widely implemented in virtual market. In this form of business arrangement the supplier places goods at a retailer's location without receiving payment, until the goods are sold. We consider a single period supply chain model, where the supplier contracts with the retailer with some probability of return. Market demand is additive, linearly price–dependent and uncertain. We focus on vendor managed consignment inventory (VMCI) channel, in which the supplier decides the consignment price and his service level and the retailer chooses the retail price. We study channel performance under VMCI setting by analysing how the model parameters influence decision quantities, channel profit and risk function. We also illustrate the obtained results by a numerical example, which explains the overall solutions well.

Key words: stochastic demand, supply chain, consignment, operations management.

1. Introduction

Supply chain management has been one of the major tasks for management professionals. The top practice for reducing the inventory cost is using the consignment, which is shifting the inventory ownership to the suppliers. The consignment is the process of placing goods in the retailer's location and no payment is made to the supplier before the item is sold. Hence, the retailer faces lower risk associated with uncertain demand, since he has no money tied up in inventory. This arrangement is called vendor managed consignment inventory (VMCI). The pioneer of VMCI is Wal-Mart, but it is applied also by on-line market places such as Amazon.com or eBay.com (Li, Zhu and Huang (2009)).

The aim of our paper is to study the decisions and channel performance for VMCI contract with additive random demand. We ask the questions how the firms interact in the channel and how system parameters affect channel performance. We focus on the type of VMCI setting, which is introduced in Ru and Wang (2010) The authors of this paper build a newsvendor type game–theoretic model to capture the connections between the supplier and the retailer, when the supplier controls the supply channel inventory. Market demand for the product is random and price–sensitive. Both the supplier and the retailer incur a linear cost for producing and

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handling the product. The supplier offers the consignment price charged to the retailer before the demand is realized and decides on the inventory level. At the same time, the retailer chooses the retail price. In Ru and Wang (2010) the authors adopted multiplicative demand function, which let them give the precise solutions. The recent paper considering similar VMCI arrangement is Hu, Chen and Yu (2015), where the variability of the channel performance with respect to the changes of the model parameters is studied. The authors use known expressions obtained in Ru and Wang (2010) and consider the uncertainty in customer return. They also discuss how the probability of return influences the price, inventory decisions and profit function. Moreover, they calculate a semi-deviation profit as a risk measure. Our study is similar to those given in Hu, Chen and Yu (2015), but it is done for the additive demand. We continue the subject of Bieniek (2017), where the precise solutions for the optimal channel components are calculated. Now we use a slightly different model with the probability of return. The findings for the risk function give important insights for the retailer or the supplier to consider, when implementing the consignment contract can give some benefits in expected profit. Furthermore, we derive for additive demand the risk measure based on a semi-deviation of profit. Finally, we analyse the numerical example using uniform distribution.

In the following we provide a review of the papers that are closely related to our study. In the paper Lee and Chu (2005) the authors present the consignment contract with inventory ownership and try to answer to the question who should control the supply chain: the supplier or the retailer. In the consignment settings studied in that paper the wholesale and the consignment prices are exogenously given. In Wang, Jiang and Shen (2004) the authors consider the consignment contract with revenue sharing between the supplier and the retailer. In this consignment arrangement the supplier retains ownership of the inventory bearing all risk. The retailer specifies the percentage allocation of sales revenue and, at the same time, the supplier chooses the product quantity and the retail price. In their study they use an iso-elastic random and price dependent demand. In Hu, Li and Govindan (2014) the return policies are added to the VMCI contract. Lately, in Olah et al. (2017) VMCI arrangement and it's benefits for the supply chain participants are widely described.

In the summary our paper contributes to the literature by: 1. considering uncertain return behaviour in VMCI contract in an additive random demand framework 2. investigating the risk analysis 3. figuring out how the uncertainty of customer return and other model parameters influence the decision variables, profit function and risk function 4. deriving managerial insights for expected profit and risk aspect. The paper is organized as follows. In Section 2 we present general assumptions and recall the definitions of the notions used in the subsequent analysis. In Section 3 we consider the performance of the decentralized channel under VMCI contract. In this section we recall the main statements of Bieniek (2017) and then we study the variability of the obtained results analytically. Section 4 is devoted to a numerical example and finally Section 5 concludes the paper.

2. General assumptions

We consider a single–period supply chain, in which the supplier (vendor) produces and sells a product to the retailer. The supplier decides his consignment price w, charged to the retailer for each unit sold. The retailer chooses the retail price p for selling the product to consumers. Denote by c_s the supplier's unit production cost and by c_r the retailer's unit handling cost. Also define $C = c_s + c_r$ as the total unit cost for channel and $\alpha = c_r/C$ as the share of channel cost that is incurred by the retailer. We assume no product shortage costs. The market demand is defined by $D(p, \varepsilon) = a - bp + \varepsilon$, where $a, b > 0$. Here ε is a continuous random variable with the expected value μ, the cumulative distribution function $F(.)$ and the probability distribution function $f(.)$, which are defined on the support $[A, B]$, where $A < 0$ and $B > 0$. The general assumptions are as follows: $C < p < p_{max}$, where $p_{max} = max_{p:a-bp+A>0} = \frac{A+a}{b}$ (cf. Rubio-Herrero, Baykal-Gursoy and Jaskiewicz (2015)) and $A + a - bC > 0$. The assumptions guarantee that realization of the demand $D(p, \varepsilon)$ is positive. The linear additive demand has different features than the iso-price-elastic demand, studied in other papers cited in our study. The additive model does not preserve iso-price-elasticity property. Its price elasticity index of the expected demand is given by $bp/(\mu + a - bp)$. The price-elasticity index is for linear demand increasing in b at any price p, so one can consider the parameter b as a surrogate of the price elasticity index like for instance in Wang, Jiang and Shen (2004).

We add consumer returns to the model studied in Bieniek (2017) and assume that the product is returned with the probability k (cf. Mostard and Teunter (2006)). The customer gets a generous full refund. At the end of the selling season the retailer pays the supplier based on the net sale, which is equal to total sales minus total returns. It appears that customer returns are very often in the e-shops. It happens for several reasons. First of all when buying via Internet the customers do not see physically the product, which often turns out to be in the wrong size or colour. Additionally, in the virtual market customers make the decision very quickly without taking it under deep consideration. But according to the legislation in many countries it is allowed to return the purchased product without giving any reason in a given time (eg two weeks).

We use the following expressions. Let c be such that $c = \frac{C}{1-k}$. Also let $z = Q - (a - bp)$ and $\mu(z) = \mu + \int_z^B (z - u)f(u)du$. As indicated in Petruzzi and Dada (1999) the quantity z can be interpreted as a safety stock since for the selected value of z we face shortages if $z < \varepsilon$ or leftovers if $z > \varepsilon$. Also z corresponds to a customer service level given by $P(D(p, \varepsilon) \leq Q) = P(\varepsilon \leq Q - (a - bp) = z) = F(z)$. The variability of the function $\mu(z)$ is important for the following analysis. Note that $\frac{d\mu(z)}{dz} = 1 - F(z)$, $\mu(.)$ is increasing in $z \in [A, B]$, $\mu(A) = A < 0$ and $\mu(B) = \mu$. We need also the lost sales rate elasticity (LSR) concept. This notion is provided by Kocabiyikoglu and Popescu (2011) and it is the percentage change in the rate of lost sales with respect to the percentage change in the price for a given quantity. The LSR elasticity combines the relative sensitivity of the lost sales with respect to its underlying factors, the price and the inventory.

Definition 1. *The LSR elasticity for a given price $p(z)$ and a service level z for additive price–dependent demand is defined as*

$$\kappa(p(z),z) = \frac{bp(z)f(z)}{1 - F(z)}.$$

3. Channel performance under VMCI contract

3.1. General solutions for VMCI

Some statements given in this subsection are proved in Bieniek (2017). The novelty is the introduction of the probability of return to the model and the derivation of expressions for the risk measure.

Under VMCI contract, the supplier makes the decisions and he is the owner of the goods until they are sold, but physically they are stored in the retailer's location. An illustrative example of VMCI is as follows. Company S (supplier) provides his product on consignment and it is responsible for manufacturing costs. Under consignment, company S is no longer responsible for material handling. From now the retailer has responsibility for any damage of goods on their properties. Company S makes the inventory decision and sets the consignment price. The retailer is able to choose the retailer price. The supplier is favoured in VMCI contract and he is the Stackelberg leader, and hence he obtains more profit. There are three steps in VMCI setting with return. Namely, in Step 1 the supplier chooses the order quantity and the consignment price charged for the retailer for each unit sold. In Step 2 the retailer decides the retail price. In Step 3 if some consumers return the product, then the retailer pays the supplier based on net sales.

The optimization problem is addressed following backward induction. First we focus on the retailer's optimization part (Step 2) and then on the supplier's optimization problem (Step 1). In Step 2 for a given consignment price and a given service level, chosen by the supplier in Step 1, the retailer determines the retail price, which maximizes his own expected profit given by

$$\Pi_R(p|w,z) = (p - w)(1 - k)E(\min(D(p,\varepsilon),Q)) - C\alpha Q.$$

This expression is equivalent to

$$\Pi_R(p|w,z) = (1 - k)\{(p - w)(\mu(z) + a - bp) - c\alpha(z + a - bp)\}.$$

The total sales of the returned items is equal to $(1 - k)E(\min(D(p,\varepsilon),Q))$. Assuming that the retailer does not get profit from the returned items, he can obtain $p - w$ from the each net sale. Under the above assumptions in Step 2 for any given service level $z \in [A, B]$ and a consignment price $w > 0$, the retailer's unique optimal retail price $p_d^*(w,z)$ is given by

$$p^*(w,z) = \frac{\mu(z) + a + bc\alpha + bw}{2b}. \tag{1}$$

In Step 1, knowing that the retailer's optimal price p^* is determined by (1), the supplier's aim is to set the optimal consignment price w^* and the optimal service level z^*, which maximize his own expected profit. The supplier's expected profit is equal to

$$\Pi_S(w,z|p) = (1-k)\{w(\mu(z)+a-bp) - c(1-\alpha)(z+a-bp)\}.$$

It is proved, that for any given service level z, the supplier's unique optimal consignment price $w^*(z)$ maximizing $\Pi_S(w,z|p)$ has the form

$$w^*(z) = \frac{\mu(z)+a+bc(1-2\alpha)}{2b}. \tag{2}$$

If LSR elasticity satisfies

$$\kappa(w^*(z)+c(1-\alpha),z) \geq \frac{1}{2} \quad \text{for any} \quad z \in [A,B], \tag{3}$$

then the service level z^* is uniquely determined by

$$\frac{\mu(z^*)+a-4bc\alpha+3bc}{2b} = \frac{2c(1-\alpha)}{1-F(z^*)}.$$

Putting the formula (2) into (1) we get $p^*(z^*) = \frac{3\mu(z^*)+3a+bc}{4b}$. Thus the total channel expected profit is equal to

$$\Pi^* = \Pi_R^* + \Pi_S^* = (1-k)\left\{\frac{3\mu(z^*)+3a+bc}{16b}(\mu(z^*)+a+3bc) - c(z^*+a)\right\}.$$

We see that (3) is equivalent to $f(z^*)(\mu(z^*)+a+3bc-4bc\alpha) - (1-F(z^*)) > 0$. Moreover, under the general assumptions $p^*(z^*) \leq \frac{3\mu(B)+3a+bc}{4b} \leq \frac{A+a}{b}$, which gives $4A+a-bc-3\mu > 0$.

In our model a supply chain bears demand risk. We consider 1-st central-semi-deviation of profit as a risk measure defined in Ogryczak and Ruszczynski (2001) and given by $\delta = E(max(\Pi-\pi),0)$, where π is a random profit and Π is the expected profit. We figure out the risk sharing by looking at $\delta_R = Emax(\Pi_R-\pi_R,0)$. It is known that $\pi_R = (1-k)((p-w)(min(\varepsilon,z)+a-bp) - c\alpha(z+a-bp))$, which implies

$$\delta_R^* := \delta_R(p^*,w^*,z^*) = (1-k)(p^*-w^*)\int_A^{\mu(z^*)}(\mu(z^*)-\varepsilon)f(\varepsilon)d\varepsilon.$$

Similarly we obtain, $\delta^* = \delta_R^* + \delta_S^* = (1-k)p^*\int_A^{\mu(z^*)}(\mu(z^*)-\varepsilon)f(\varepsilon)d\varepsilon$. Then the retailer share of channel risk defined by $\beta = \frac{\delta_R^*}{\delta^*}$ for additive demand is equal to

$$\beta = \frac{p^*-w^*}{p^*} = \frac{\mu(z^*)+a-bc+4bc\alpha}{3\mu(z^*)+3a+bc}.$$

3.2. Sensitivity analysis of VMCI channel

Now we analyse how the model parameters influence the channel performance. All proofs of the propositions from this subsection are given in Appendix.

Proposition 1. *If* (3) *holds then z^* increases in α and for $\alpha > \frac{1}{2}$ it decreases in b, k and C.*

The optimal service level z^* increases if the share of channel cost increases and then the retailer shares larger part of channel cost than the supplier. If the price elasticity increases then the service level decreases. Moreover, when uncertainty increases, the amount of the returned items increases and the net sale decreases. Consequently, the decision maker has to stock less of an item.

Proposition 2. *If* (3) *holds then p^* increases in α and it decreases in b. The monotonicity of p^* in k and C depends on other parameters.*

The optimal retail price is increasing with the share of channel cost and decreasing with the price elasticity, which is the same as the conclusions for the service level. But the variability with respect to the probability of return and the total cost depends on other parameters.

Proposition 3. *If* (3) *holds then the monotonicity of w^* in α depends on other parameters and w^* decreases in b. Moreover, if $\alpha > \frac{1}{2}$ then w^* decreases in C and k, otherwise it's monotonicity depends on other parameters.*

The conclusion for the consignment price regarding the influence of the price sensitivity is similar to those obtained for the retail price.

Proposition 4. *If*

$$\kappa \left(w^*(z) + c \left(\alpha + \frac{1}{3} \right), z \right) \geq \frac{1}{2} \quad \text{for any} \quad z \in [A, B], \tag{4}$$

and (3) *holds, then Π^* is increasing-decreasing both in z^* and α.*

Remark 1. *The constraint* (4) *is equivalent to $(\mu(z) + a + \frac{5}{3})f(z) \geq (1 - F(z))$, which is independent on α.*

The crucial part of the above proposition is that the optimal channel profit is first an increasing function and then a decreasing function of z^*. Other statements are the consequence of that property. This is different than for the multiplicative model, where the channel profit always increases in z. It should be underlined that for additive demand the propositions hold only for selected values of model parameters, which satisfy (3) and (4) due to the very demanding assumption of the profit concavity. Finding the milder assumptions can be the topic of a future research.

Proposition 5. *If* (3) *holds then β increases in z^* for $\alpha < \frac{1}{3}$ and it decreases otherwise. Moreover, it increases in α for $\alpha < \frac{1}{3}$ and it's monotonicity depends on other parameters otherwise.*

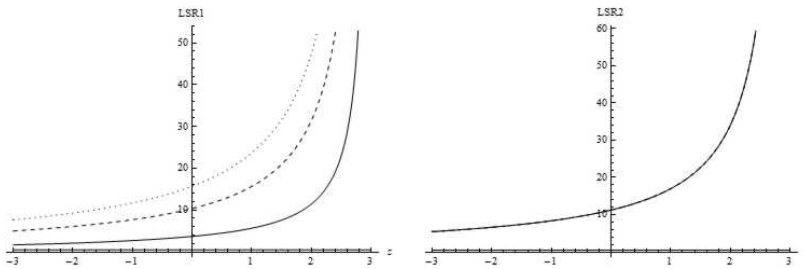Figure 1: LSR elasticity $\kappa(w^*(z)+c(1-\alpha),z)$ for $\alpha=0.9$ (solid), $\alpha=0.4$ (dotted) and $\alpha=0.1$ (dashed) (left) and $\kappa(w^*(z)+c(\alpha+1/3),z)$ (right) with respect to z

From the above statement we see that a higher risk - higher expected profit phenomenon does not always apply to the retailer and it depends critically on his share of channel cost. For $\alpha<\frac{1}{3}$ the higher retailer share of risk corresponds to the higher share of channel cost or equivalently to the lower retailer's profit.

4. Numerical example

In order to illustrate the results previously obtained we proceed with a numerical example. We use uniformly distributed demand on the interval $[-3,3]$. Moreover, we set the base values equal to $a=35$, $b=1$, $C=10$, $k=0.5$ (then $c=20$). Using the figures we present how the model parameters influence the optimal solutions. We investigate the variability in one parameter with other base parameters. All figures justify the statements of the propositions. First we state that α should be such that (3) and (4) are satisfied. This can be confirmed by the fact, that the figures of the elasticities should lie above the line $y=\frac{1}{2}$ (Fig. 1).

Figure 2 shows the variability of the service level. In the figure on the left we can see that the service level increases in α. In the figure of the right we observe that if b changes from 0.1 to 1.2 then the service level decreases for different values of α. Figure 3 presents the variability of the retail price. We see that the retail price increases if α increases and decreases in b. Finally, on Figure 4 it is shown that the optimal expected profit is increasing-decreasing in α and it decreases in b.

5. Conclusions

In this paper we study the vendor managed inventory contract with consignment (VMCI), which is widely applied in many industries, including virtual market. In VMCI arrangement the upstream supplier owns the product until it is sold by the downstream retailer. The supplier decides on the consignment price and the inventory level. The retailer chooses the retail price.

Our study is mainly based on Ru and Wang (2010), Hu, Chen and Yu (2015) and Bieniek (2017). VMCI program studied by Ru and Wang (2010) uses multiplicative
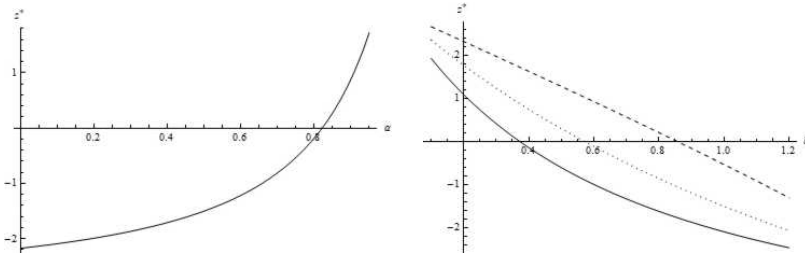
Figure 2: Service level with respect to α (left) and b for $\alpha = 0.1$ (solid), $\alpha = 0.5$ (dotted) and $\alpha = 0.75$ (dashed) (right)
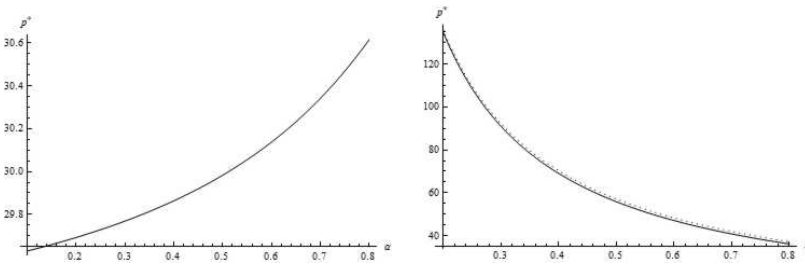

Figure 3: Price with respect to α (left) and b for $\alpha = 0.1$ (solid), $\alpha = 0.75$ (dotted) (right)
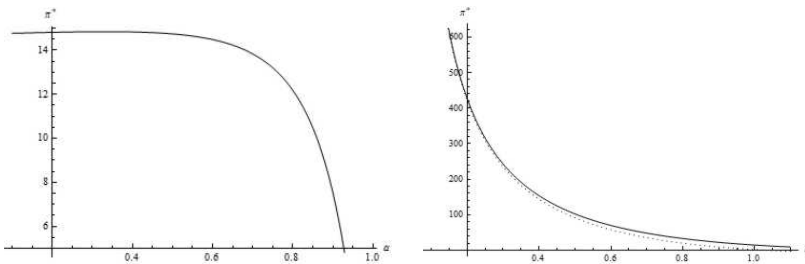

Figure 4: Profit with respect to α (left) and b for $\alpha = 0.05$ (solid), $\alpha = 0.9$ (dotted) (right)

random demand, which is exponentially dependent on the price. The additive random demand linearly dependent on the price is considered in Bieniek (2017). In that paper the precise formulas are given, but the sensitivity of the optimal solutions is not examined. We consider this sensitivity in our paper, where also an opportunity of return with a given return probability is added to the model. Our work is done in the same light as in Hu, Chen and Yu (2015), but for additive demand form. Precisely, we show how the share of channel cost, probability of return and other parameters influence the price, inventory decisions and profit function. We discuss also the retailer share of channel risk and it's dependence on the share of channel cost. We use the central-semi deviation risk measure discovered by Ogryczak and Ruszczynski (2001). The assumptions involve the lost sales rate elasticity, which has an economic interpretation. In the last part we consider the numerical example. We decide to present rather figures than tables since they illustrate the results more clearly. We use uniformly distributed demand with mean zero and the model parameters satisfying all of the assumptions. It can be seen that the presented figures justify the statements of the established propositions.

Our research proves that the form of the demand influences on the supply chain performance. Our findings for risk function give important insights for the retailer or the supplier to consider, when implementing consignment contract can give some benefits in the expected profit. As a future research the considerations on models with multiple suppliers or competing retailers could be done.

6. Appendix

Proof.[of Proposition 1] From (3.1) we have $G(\alpha, z^*) = 0$, where $G(\alpha, z^*) = (\mu(z^*) + a - 4bc\alpha + 3bc)(1 - F(z^*)) - 4bc(1 - \alpha)$. By implicit function theorem we get $\frac{dG}{d\alpha} + \frac{dG}{dz^*}\frac{dz^*}{d\alpha} = 0$, which gives

$$\frac{dz^*}{d\alpha} = \frac{4bcF(z^*)}{f(z^*)(\mu(z^*) + a + 3bc - 4bc\alpha) - (1 - F(z^*))^2},$$

which by (3) is positive. Now let $m = bc = \frac{bC}{1-k}$. Then $G(m, z^*) = (\mu(z^*) + a + m(3 - 4\alpha)(1 - F(z^*)) - 4m(1 - \alpha)$ and by implicit function theorem we get

$$\frac{dz^*}{dm} = \frac{1 + (3 - 4\alpha)F(z^*)}{-f(z^*)(\mu(z^*) + a + 3m - 4m\alpha) + (1 - F(z^*))^2},$$

which by (3) is negative. The proof is complete. □

Proof.[of Propositions 2, 3, 5] The proofs can be conducted analogously to the proof of Proposition 1 using the standard calculus. □

Proof.[of Proposition 4] Let $\Pi_d = \frac{1}{1-k}\Pi^*$. Then $\frac{d\Pi_d}{dz^*} = \frac{1-F(z^*)}{8b}(3\mu(z^*) + 3a + 5bc) - c$. Moreover, $\frac{d\Pi_d}{dz^*}|_A = \frac{3}{8b}(A + a - bc) > 0$ and $\frac{d\Pi_d}{dz^*}|_B = -c < 0$, which implies that there exists z_d such that $\frac{d\Pi_d}{dz_d} = 0$. A sufficient condition for uniqueness is that the second

derivative of $\Pi_d(z)$ should be negative, which is true since by (4)

$$\frac{d^2\Pi_d}{dz^2} = -\frac{1-F(z)}{8b}\left\{\frac{f(z)}{1-F(z)}(3\mu(z)+3a+5bc)-3(1-F(z))\right\} < 0.$$

We state that Π_d depends on α only by z thus it is increasing-decreasing in α. The same conclusions hold for Π^*. The proof is complete.

□

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