STATISTICS IN TRANSITION new series, Spring 2014
Vol. 15, No. 2, pp. 197–220

EFFECTIVE ROTATION PATTERNS FOR MEDIAN ESTIMATION IN SUCCESSIVE SAMPLING

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ABSTRACT

The present work deals with the problem of estimation of population median at current occasion in two-occasion successive sampling. Best linear unbiased estimators have been proposed by utilizing additional auxiliary information, readily available on both the occasions. Asymptotic variances of the proposed estimators are derived and the optimum replacement policies are discussed. The behaviours of the proposed estimators are analyzed on the basis of data from natural populations. Simulation studies have been carried out to measure the precision of the proposed estimators.

Key words: population median, successive sampling, auxiliary information, optimum replacement policy.

1. Introduction

When the value of the study character of a finite population is subject to change (dynamically) over time, a survey carried out on a single occasion will provide information about the characteristics of the surveyed population for the given occasion only and will not give any information on the nature of change of the characteristic over different occasions and the average value of the characteristic over all occasions or the most recent occasion. To meet these requirements, sampling is done on successive occasions that provide a strong tool for generating the reliable estimates at different occasions. The problem of sampling on two successive occasions was first considered by Jessen (1942), and later this idea was extended by Patterson (1950), Narain (1953), Eckler (1955), Gordon (1983), Arnab and Okafor (1992), Feng and Zou (1997), Singh and Singh (2001), Singh and Priyanka (2008), Singh et al. (2012), Bandyopadhyay and Singh (2014), and many others.

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All the abovestudies were concerned with the estimation of population mean or variance on two or more occasions.

There are many problems of practical interest which involves variables with extreme values that strongly influence the value of the mean. In such situations the study variable is having highly skewed distributions. For example, the study of environmental issues, the study of social evil such as abortions, the study of income, expenditure, etc. In these situations, the mean may offer results which are not representative enough because the mean moves with the direction of the asymmetry. The median, on the other hand, is unaffected by extreme values.

Most of the studies related to medians have been developed by assuming simple random sampling or its ramification in stratified random sampling (Gross (1980), Sedransk and Meyer (1978), Smith and Sedransk (1983) consider only the variable of interest without making explicit use of auxiliary variables. Some of the researchers, namely Chambers and Dunstan (1986), Kuk and Mak (1989), Rao et al. (1990), Rueda et al.(1998), Khoshnevisan et al. (2002), Singh and Solanki (2013) etc., make use of auxiliary variables to estimate the population median).

It is to be mentioned that a large number of estimators for estimating the population mean at current occasion have been proposed by various authors, however only a few efforts (namely Martinez-Miranda et al. (2005), Singh et al. (2007), Rueda et al. (2008) and Gupta et al. (2008)) have been made to estimate the population median on the current occasion in two occasions successive sampling. It is well known that the use of auxiliary information at the estimation stage can typically increase the precision of estimates of a parameter. To the best of our knowledge, no effort has been made to use additional auxiliary information readily available on both the occasions to estimate population median at current occasion in two-occasion successive sampling.

Motivated with the above arguments and utilizing the information on an additional auxiliary variable, readily available on both the occasions, the best linear unbiased estimators for estimating the population median on current occasion in two-occasion successive sampling have been proposed. It has been assumed that the additional auxiliary variable is stable over the two-occasions.

The paper is spread over ten sections. Sample structure and notations have been discussed in section 2. In section 3 the proposed estimator has been formulated. Properties of proposed estimators including variances are derived under section 4. Minimum variance of the proposed estimator is derived in section 5. Practicability of the proposed estimator is also discussed. In section 6 optimum replacement policies are discussed. Section 7 contains comparison of the proposed estimator with the natural sample median estimator when there is no matching from the previous occasion and the estimator when no additional auxiliary information has been used. Practicability of the estimator Δ is also discussed. In section 8 simulation studies have been carried out to investigate the performance of the proposed estimators. The results obtained as a result of empirical and simulation studies have been elaborated in section 9. Finally, the conclusion of the entire work has been presented in section 10.

2. Sample structures and notations

Let $U = (U_1, U_2, ---, U_N)$ be the finite population of N units, which has been sampled over two occasions. It is assumed that the size of the population remains unchanged but values of the unit change over two occasions. Let the character under study be denoted by x (y) on the first (second) occasion respectively. It is further assumed that information on an auxiliary variable z (with known population median) is available on both the occasions. A simple random sample (without replacement) of n units is taken on the first occasion. A random sub-sample of $m = n\lambda$ units is retained (matched) for use on the second occasion. Now, at the current occasion a simple random sample (without replacement) of $u = (n - m) = n\mu$ units is drawn afresh from the remaining $(N - n)$ units of the population so that the sample size on the second occasion is also n. λ and μ, $(\lambda + \mu = 1)$ are the fractions of matched and fresh samples respectively at the second (current) occasion. The following notations are considered for further use:

M_x, M_y, M_z : Population median of x, y and z, respectively.

$\hat{M}_{x(n)}$, $\hat{M}_{x(m)}$, $\hat{M}_{y(m)}$, $\hat{M}_{y(u)}$, $\hat{M}_{z(n)}$, $\hat{M}_{z(m)}$, $\hat{M}_{z(u)}$: Sample median of the respective variables of the sample sizes shown in suffices.

ρ_{yx}, ρ_{xz}, ρ_{yz} : The Correlation coefficient between the variables shown in suffices.

3. Formulation of estimator

To estimate the population median M_y on the current (second) occasion, the minimum variance linear unbiased estimator of M_y under SRSWOR sampling scheme have been proposed and is given as

$$T = \left\{\alpha_1 \hat{M}_{y(u)} + \alpha_2 \hat{M}_{y(m)}\right\} + \left\{\alpha_3 \hat{M}_{x(m)} + \alpha_4 \hat{M}_{x(n)}\right\} + \left\{\alpha_5 \hat{M}_{z(u)} + \alpha_6 \hat{M}_{z(m)} + \alpha_7 \hat{M}_{z(n)} + \alpha_8 M_z\right\} \tag{1}$$

where $\alpha_i \left(i = 1,\ 2,\ ---,\ 8\right)$ are constants to be determined so that

(i) The estimator T becomes unbiased for M_y and

(ii) The variance of T attains a minimum

For unbiasedness, the following conditions must hold:

$\left(\alpha_1 + \alpha_2\right) = 1$, $\left(\alpha_3 + \alpha_4\right) = 0$ and $\left(\alpha_5 + \alpha_6 + \alpha_7 + \alpha_8\right) = 0$.

Substituting $\alpha_1 = \phi_1, \alpha_3 = \beta_1$ and $\alpha_8 = -(\alpha_5 + \alpha_6 + \alpha_7)$ in equation (1), the estimator *T* takes the following form:

$$T = \left\{\phi_1 \hat{M}_{y(u)} + (1-\phi)\hat{M}_{y(m)}\right\} + \beta_1\left\{\hat{M}_{x(m)} - \hat{M}_{x(n)}\right\} + \left\{\alpha_5\left(\hat{M}_{z(u)} - M_z\right) + \alpha_6\left(\hat{M}_{z(m)} - M_Z\right) + \alpha_7\left(\hat{M}_{z(n)} - M_Z\right)\right\}$$

$$= \phi_1\left\{\hat{M}_{y(u)} + k_1\left(\hat{M}_{z(u)} - M_Z\right)\right\} + (1-\phi_1)\left\{\hat{M}_{y(m)} + k_2\left(\hat{M}_{x(m)} - \hat{M}_{x(n)}\right) + k_3\left(\hat{M}_{z(m)} - M_Z\right) + k_4\left(\hat{M}_{z(n)} - M_Z\right)\right\}$$

$$T = \phi_1 T_1 + (1-\phi_1)\, T_2 \tag{2}$$

where $T_1 = \hat{M}_{y(u)} + k_1\left(\hat{M}_{z(u)} - M_Z\right)$ is based on the sample of size *u* drawn afresh at current occasion and the estimator

$$T_2 = \left\{\hat{M}_{y(m)} + k_2\left(\hat{M}_{x(m)} - \hat{M}_{x(n)}\right) + k_3\left(\hat{M}_{z(m)} - M_Z\right) + k_4\left(\hat{M}_{z(n)} - M_Z\right)\right\}$$

is based on the sample of size *m* matched form previous occasion.

$k_1 = \frac{\alpha_5}{\phi_1}$, $k_2 = \frac{\beta_1}{1-\phi_1}$, $k_3 = \frac{\alpha_6}{1-\phi_1}$, $k_4 = \frac{\alpha_7}{1-\phi_1}$ and ϕ_1 are the unknown constants to be determined so as to minimize the variance of estimator *T*.

Remark 3.1. For estimating the median on each occasion, the estimator T_1 is suitable, which implies that more belief on T_1 could be shown by choosing ϕ_1 as 1 (or close to 1), while for estimating the change from one occasion to the next, the estimator T_2 could be more useful so ϕ_1 be chosen as 0 (or close to 0). For asserting both the problems simultaneously, the suitable (optimum) choice of ϕ_1 is required.

4. Properties of the estimator *T*

The properties of the proposed estimator *T* are derived under the following assumptions:

(i) Population size is sufficiently large *(i.e. N→∞),* therefore finite population corrections are ignored.

(ii) As *N→∞,* the distribution of bivariate variable *(a, b)* where *a* and *b* $\in \{x, y, z\}$ and $a \neq b$ approaches a continuous distribution with marginal densities $f_a(\cdot)$ and $f_b(\cdot)$ for a and b respectively, see Kuk and Mak (1989).

(iii) The marginal densities $f_x(\cdot)$, $f_y(\cdot)$ and $f_z(\cdot)$ are positive.

(iv) The sample medians $\hat{M}_{x(n)}$, $\hat{M}_{x(m)}$, $\hat{M}_{y(m)}$, $\hat{M}_{y(u)}$, $\hat{M}_{z(n)}$, $\hat{M}_{z(m)}$ and $\hat{M}_{z(u)}$ are consistent and asymptotically normal (see Gross (1980)).

(v) Following Kuk and Mak (1989), let P_{ab} be the proportion of elements in the population such that $a \le M_a$ and $b \le M_b$ where a and $b \in \{x, y, z\}$ and $a \neq b$.

(vi) The following large sample approximations are assumed:

$\hat{M}_{y(u)} = M_y(1+e_0)$, $\hat{M}_{y(m)} = M_y(1+e_1)$, $\hat{M}_{x(m)} = M_x(1+e_2)$, $\hat{M}_{x(n)} = M_x(1+e_3)$, $\hat{M}_{z(u)} = M_z(1+e_4)$, $\hat{M}_{z(m)} = M_z(1+e_5)$ and $\hat{M}_{z(n)} = M_z(1+e_6)$ such that $|e_i| < 1$ $\forall\ i = 0, 1, 2, 3, 4, 5, 6$.

The values of various related expectations can be seen in Allen et al. (2002) and Singh (2003). Under the above transformations, the estimators T_1 and T_2 take the following forms:

$$T_1 = M_y(1+e_0) + k_1 M_z e_4 \tag{3}$$

$$T_2 = M_y(1+e_1) + k_2 M_x(e_2 - e_3) + M_z(k_3 e_5 + k_4 e_6) \tag{4}$$

Thus we have the following theorems:

Theorem 4.1. T is unbiased estimator of M_y.

Proof: Since T_1 and T_2 are difference and difference-type estimators, respectively, they are unbiased for M_y. The combined estimator T is a convex linear combination of T_1 and T_2, hence it is also an unbiased estimator of M_y.

Theorem 4.2. Ignoring the finite population corrections, the variance of T is

$$V(T) = \phi_1^2\, V(T_1) + (1 - \phi_1)^2 V(T_2) \tag{5}$$

where
$$V(T_1) = \frac{1}{u}\xi_1 \tag{6}$$

and
$$V(T_2) = \frac{1}{m}\xi_2 + \left(\frac{1}{m} - \frac{1}{n}\right)\xi_3 + \frac{1}{n}\xi_4 \tag{7}$$

$$\xi_1 = A_1 + k_1^2 A_2 + 2k_1 A_3, \xi_2 = A_1 + k_3^2 A_2 + 2k_3 A_3,$$
$$\xi_3 = k_2^2 A_4 + 2k_2 A_5 + 2k_2 k_3 A_6, \xi_4 = k_4^2 A_2 + 2k_4 A_3 + 2k_3 k_4 A_2,$$
$$A_1 = \frac{1}{4}\left\{ f_y\left(M_y\right)\right\}^{-2}, A_2 = \frac{1}{4}\left\{ f_z\left(M_z\right)\right\}^{-2},$$
$$A_3 = \left(P_{yz} - 0\cdot 25\right)\left\{ f_y\left(M_y\right)\right\}^{-1}\left\{ f_z\left(M_z\right)\right\}^{-1}, A_4 = \frac{1}{4}\left\{ f_x\left(M_x\right)\right\}^{-2},$$
$$A_5 = \left(P_{yx} - 0\cdot 25\right)\left\{ f_y\left(M_y\right)\right\}^{-1}\left\{ f_x\left(M_x\right)\right\}^{-1} \text{ and}$$
$$A_6 = \left(P_{xz} - 0\cdot 25\right)\left\{ f_x\left(M_x\right)\right\}^{-1}\left\{ f_z\left(M_z\right)\right\}^{-1}.$$

Proof: The variance of T is given by

$$V\left(T\right) = E\left(T - M_y\right)^2 = E\left[\phi_1\left(T_1 - M_y\right) + \left(1 - \phi_1\right)\left(T_2 - M_y\right)\right]^2$$
$$= \phi_1^2\ V\left(T_1\right) + \left(1 - \phi_1\right)^2 V\left(T_2\right) + \phi_1\left(1 - \phi_1\right)\ \mathrm{cov}\left(T_1, T_2\right) \tag{8}$$

where $V\left(T_1\right) = E\left(T_1 - M_y\right)^2$ and $V\left(T_2\right) = E\left(T_2 - M_y\right)^2$.

As T_1 and T_2 are based on two independent samples of sizes *u* and *m* respectively, hence $\mathrm{cov}\ \left(T_1, T_2\right) = 0$.

Now, substituting the expressions of T_1 and T_2 from equations (3) and (4) in equation (8), taking expectations and ignoring finite population corrections, we have the expression for variance of *T* as in equation (5).

5. Minimum variance of the estimator *T*

Since the variance of the estimator *T* in equation (5) is the function of unknown constants $k_1,\ k_2,\ k_3,\ k_4$ and ϕ_1, therefore it is minimized with respect to $k_1,\ k_2,\ k_3,\ k_4$ and ϕ_1 and subsequently the optimum values of $k_1,\ k_2,\ k_3,\ k_4$ and ϕ_1 are obtained as

$$k_1^* = \frac{-A_3}{A_2} \tag{9}$$

$$k_2^* = \frac{A_3 A_4 A_6 - A_2 A_4 A_5}{A_4\left(A_2 A_4 - A_6^2\right)} \tag{10}$$

$$k_3^* = \frac{-A_3 A_4 + A_5 A_6}{\left(A_2 A_4 - A_6^2\right)} \tag{11}$$

$$k_4^* = \frac{A_3 A_6^2 - A_2 A_5 A_6}{A_2 \left(A_2 A_4 - A_6^2 \right)} \tag{12}$$

$$\phi_{1opt.} = \frac{V\left(T_2\right)}{V\left(T_1\right) + V\left(T_2\right)} \tag{13}$$

Using the optimum values of k_i's $\left(i = 1,\ 2,\ 3,\ 4\right)$ in equation (6) and (7), we get the optimum variances of T_1 and T_2 as

$$V\left(T_1\right)_{opt.} = \frac{1}{u} A_7 \tag{14}$$

$$V\left(T_2\right)_{opt.} = \frac{1}{m} A_8 + \left(\frac{1}{m} - \frac{1}{n}\right) A_9 + \frac{1}{n} A_{10} \tag{15}$$

where $A_7 = A_1 + k_1^{*2} A_2 + 2k_1^* A_3$, $A_8 = A_1 + k_3^{*2} A_2 + 2k_3^* A_3$

$$A_9 = k_2^{*2} A_4 + 2k_2^* A_5 + 2k_2^* k_3^* A_6 \quad \text{and}$$

$$A_{10} = k_4^{*2} A_2 + 2k_4^* A_3 + 2k_3^* k_4^* A_2 .$$

Further, substituting the values of $V\left(T_1\right)_{opt.}$ and $V\left(T_2\right)_{opt.}$ from equations (14) and (15) in equation (13), we get the optimum values of $\phi_{1opt.}$ with respect to k_i^*'s $\left(i = 1,\ 2,\ 3,\ 4\right)$ as

$$\phi_{1opt.}^* = \frac{V\left(T_2\right)_{opt.}}{V\left(T_1\right)_{opt.} + V\left(T_2\right)_{opt.}} \tag{16}$$

Again substituting the value of $\phi_{1opt.}^*$ from equation (16) in equation (5), we get the optimum variance of T as

$$\mathrm{V}\left(\mathrm{T}\right)_{\mathrm{opt.}} = \frac{\mathrm{V}\left(\mathrm{T}_1\right)_{\mathrm{opt.}} \mathrm{V}\left(\mathrm{T}_2\right)_{\mathrm{opt.}}}{\mathrm{V}\left(\mathrm{T}_1\right)_{\mathrm{opt.}} + \mathrm{V}\left(\mathrm{T}_2\right)_{\mathrm{opt.}}} \tag{17}$$

Further, substituting the value from (14) and (15) in equation (16) and (17), we get the simplified values of $\phi_{1opt.}^*$ and $V\left(T\right)_{opt.}$ as

$$\phi_{1opt.}^* = \frac{\mu\left(A_{11} + \mu A_{12}\right)}{\mu^2 A_{12} + \mu^2 A_{13} + A_7} \tag{18}$$

$$V\left(T\right)_{opt.} = \frac{1}{n} \frac{A_7\left(A_{11} + \mu A_{12}\right)}{\left(\mu^2 A_{12} + \mu A_{13} + A_7\right)} \tag{19}$$

where $A_{11} = A_8 + A_{10}$, $A_{12} = A_9 - A_{10}$, $A_{13} = A_{11} - A_7$ and μ is the fraction of fresh sample at current occasion for the estimator *T*.

5.1. Estimator *T* in practice

The main difficulty in using the proposed estimator *T* defined in equation (2) is the availability of $k_i's$ $(i=1, 2, 3, 4)$ as the optimum values of $k_i's$ $(i=1, 2, 3, 4)$ depend on the population parameters $P_{yx}, P_{yz}, P_{xz}, f_y(M_y), f_x(M_x)$ and $f_z(M_z)$. If these parameters are known, the proposed estimator can be easily implemented. Otherwise, which is the most often situation in practice, the unknown population parameters are replaced by their respective sample estimates. The population proportions P_{yx}, P_{yz} and P_{xz} are replaced by the sample estimates $\hat{P}_{yx}$, $\hat{P}_{yz}$ and $\hat{P}_{xz}$ respectively, and the marginal densities $f_y(M_y), f_x(M_x)$ and $f_z(M_z)$ can be substituted by their kernel estimator or nearest neighbour density estimator or generalized nearest neighbour density estimator related to the kernel estimator (Silverman (1986)). Here, the marginal densities $f_y(M_y)$, $f_x(M_x)$ and $f_z(M_z)$ are replaced by $\hat{f}_y(\hat{M}_{y(m)}), \hat{f}_x(\hat{M}_{x(n)})$ and $\hat{f}_z(\hat{M}_{z(n)})$ respectively, which are obtained by the method of generalized nearest neighbour density estimation related to the kernel estimator.

Remark 5.1.1. To estimate $f_x(M_x)$ by the generalized nearest neighbour density estimator related to the kernel estimator, the following procedure has been adopted:

Choose an integer $h \approx n^{1/2}$ and define the distance $d(x_1, x_2)$ between two points on the line to be $|x_1 - x_2|$.

For $\hat{M}_{x(n)}$ define $d_1(\hat{M}_{x(n)}) \le d_2(\hat{M}_{x(n)}) \le --- \le d_n(\hat{M}_{x(n)})$ to be the distances, arranged in ascending order, from $\hat{M}_{x(n)}$ to the points of the sample.

The generalized nearest neighbour density estimate is defined by

$$\hat{f}(\hat{M}_{x(n)}) = \frac{1}{nd_h(\hat{M}_{x(n)})} \sum_{i=1}^{n} K\left(\frac{\hat{M}_{x(n)} - x_i}{d_h(\hat{M}_{x(n)})}\right) \tag{20}$$

where the kernel function *K*, satisfies the condition $\int_{-\infty}^{\infty} K(x)\,dx = 1$.

Here, the kernel function is chosen as Gaussian Kernel given by $K(x) = \frac{1}{2\pi} e^{-\left(\frac{1}{2}x^2\right)}$.

Similarly, the estimate of $f_y(M_y)$ and $f_z(M_z)$ can be obtained.

Remark 5.1.2. For estimating $f_y\left(M_y\right)$, P_{yz} and P_{yx} we have two independent samples of sizes *u* and *m* respectively at current occasion. So, either of the two can be used, but in general for good sampling design in successive sampling $u \le m$. So, in the present work $f_y\left(M_y\right)$, P_{yz} and P_{yx} are estimated from the sample of size *m*, matched from the first occasion.

Therefore, under the above substitutions of the unknown population parameters by their respective sample estimates, the estimator *T* takes the following form:

$$T^* = \psi_1 T_1^* + \left(1 - \psi_1\right) T_2^* \tag{21}$$

where

$$T_1^* = \hat{M}_{y(u)} + k_1^{**}\left(\hat{M}_{z(u)} - M_Z\right) \tag{22}$$

and

$$T_2^* = \left\{\hat{M}_{y(m)} + k_2^{**}\left(\hat{M}_{x(m)} - \hat{M}_{x(n)}\right) + k_3^{**}\left(\hat{M}_{z(m)} - M_Z\right) + k_4^{**}\left(\hat{M}_{z(n)} - M_Z\right)\right\} \tag{23}$$

$$k_1^{**} = \frac{-A_3^*}{A_2^*},\quad k_2^{**} = \frac{A_3^* A_4^* A_6^* - A_2^* A_4^* A_5^*}{A_4^*\left(A_2^* A_4^* - A_6^{*2}\right)},\ k_3^{**} = \frac{-A_3^* A_4^* + A_5^* A_6^*}{\left(A_2^* A_4^* - A_6^{*2}\right)},$$

$$k_4^{**} = \frac{A_3^* A_6^{*2} - A_2^* A_5^* A_6^*}{A_2^*\left(A_2^* A_4^* - A_6^{*2}\right)},\ A_1^* = \frac{1}{4}\left\{\hat{f}_y\left(\hat{M}_{y(m)}\right)\right\}^{-2},\qquad A_2^* = \frac{1}{4}\left\{\hat{f}_z\left(\hat{M}_{z(n)}\right)\right\}^{-2},$$

$$A_3^* = \left(\hat{P}_{yz} - 0\cdot 25\right)\left\{\hat{f}_y\left(\hat{M}_{y(m)}\right)\right\}^{-1}\left\{\hat{f}_z\left(\hat{M}_{z(n)}\right)\right\}^{-1},\qquad A_4^* = \frac{1}{4}\left\{\hat{f}_x\left(\hat{M}_{x(n)}\right)\right\}^{-2},$$

$$A_5^* = \left(\hat{P}_{yx} - 0\cdot 25\right)\left\{\hat{f}_y\left(\hat{M}_{y(m)}\right)\right\}^{-1}\left\{\hat{f}_x\left(\hat{M}_{x(n)}\right)\right\}^{-1} \text{ and}$$

$$A_6^* = \left(\hat{P}_{xz} - 0\cdot 25\right)\left\{\hat{f}_x\left(\hat{M}_{x(n)}\right)\right\}^{-1}\left\{\hat{f}_z\left(\hat{M}_{z(n)}\right)\right\}^{-1}.$$

ψ_1 is an unknown constant to be determined so as to minimize the mean square error of the estimator T^*.

Remark 5.1.3. The proposed estimator *T* is a difference-type estimator therefore after replacing the unknown population parameters by their respective sample estimates it becomes a regression-type estimator. Hence, up to the first order of approximations the estimator T^* will be equally precise to that of the estimator *T* (see Singh and Priyanka (2008)). Therefore, similar conclusions are applicable for T^* as that of *T*.

6. Optimum replacement policy

To determine the optimum value of μ (fraction of a sample to be taken afresh at second occasion) so that M_y may be estimated with maximum precision, we minimize $V(T)_{opt\cdot}$ in equation (19) with respect to μ and hence we get the optimum value of μ as

$$\mu_{opt\cdot^*} = \frac{-S_2 \pm \sqrt{S_2^2 - S_1S_3}}{S_1} = \mu_0 \text{ (say)} \tag{24}$$

where $S_1 = A_{12}^2$, $S_2 = A_{11}A_{12}$ and $S_3 = A_{11}A_{13} - A_7A_{12}$.

From equation (24) it is obvious that the real value of $\mu_{opt\cdot}$ exists if $S_2^2 - S_1S_3 \geq 0$. For certain situation, there might be two values of $\mu_{opt\cdot}$ satisfying the above condition, hence to choose a value of $\mu_{opt\cdot}$, it should be remembered that $0 \leq \mu_{opt\cdot} \leq 1$. All other values of $\mu_{opt\cdot}$ are inadmissible. In case both the values of $\mu_{opt\cdot}$ are admissible, we choose the minimum of these two as μ_0. Substituting the value of $\mu_{opt\cdot}$ from equation (24) in (19) we have

$$V(T)_{opt\cdot^*} = \frac{1}{n}\frac{A_7\left(A_{11} + \mu_0 A_{12}\right)}{\left(\mu_0^2 A_{12} + \mu_0 A_{13} + A_7\right)} \tag{25}$$

where $V(T)_{opt\cdot^*}$ is the optimum value of T with respect μ.

7. Efficiency comparison

To study the performance of the estimator T, the percent relative efficiencies of T with respect to (i) $\hat{M}_{y(n)}$, the natural estimator of M_y, when there is no matching, and (ii) the estimator Δ, when no additional auxiliary information is used at any occasion, have been computed for two natural population data. The estimator Δ is defined under the same circumstances as the estimator T, but in the absence of information on additional auxiliary variable z on both the occasions is proposed as

$$\Delta = \left\{\delta_1 \hat{M}_{y(u)} + \delta_2 \hat{M}_{y(m)}\right\} + \left\{\delta_3 \hat{M}_{x(m)} + \delta_4 \hat{M}_{x(n)}\right\} \tag{26}$$

where $\delta_i \left(i = 1, 2, 3, 4\right)$ are constants to be determined so that

(i) The estimator Δ becomes unbiased for M_y and

(ii) The variance of Δ attains the minimum.

For unbiasedness, the following conditions must hold:
$\left(\delta_1+\delta_2\right)=1$ and $\left(\delta_3+\delta_4\right)=0$.

Substituting $\delta_1=\phi_2$ and $\delta_3=\beta_2$ in equation (26), the estimator Δ takes the following form:

$$\Delta=\left\{\phi_2\hat{M}_{y(u)}+\left(1-\phi_2\right)\hat{M}_{y(m)}\right\}+\beta_2\left(\hat{M}_{x(m)}-\hat{M}_{x(n)}\right)$$

$$=\phi_2\hat{M}_{y(u)}+\left(1-\phi_2\right)\left\{\hat{M}_{y(m)}+k_5\left(\hat{M}_{x(m)}-\hat{M}_{x(n)}\right)\right\}$$

$$\Delta=\phi_2\Delta_1+\left(1-\phi_2\right)\Delta_2 \tag{27}$$

where the estimator $\Delta_1=\hat{M}_{y(u)}$ is based on the fresh sample of size *u* and the estimator $\Delta_2=\left\{\hat{M}_{y(m)}+k_5\left(\hat{M}_{x(m)}-\hat{M}_{x(n)}\right)\right\}$ is based on the matched sample of size *m*, $k_5=\frac{\beta_2}{\left(1-\phi_2\right)}$ and ϕ_2 are the unknown constants to be determined so as to minimize the variance of estimator Δ. Following the methods discussed in Sections 4, 5 and 6, the optimum value of k_5, $\mu_{1opt.}$ (optimum value of fraction of the fresh sample for the estimator Δ), variance of $\hat{M}_{y(n)}$ and optimum variance of Δ ignoring the finite population corrections are given by

$$k_5^*=\frac{-A_5}{A_4} \tag{28}$$

$$\mu_{1opt.^*}=\frac{-A_1\pm\sqrt{A_1\left(A_1+A_{14}\right)}}{A_{14}}=\mu^*\left(say\right) \tag{29}$$

$$V\left(\hat{M}_{y(n)}\right)=\frac{1}{n}A_1 \tag{30}$$

$$V\left(\Delta\right)_{opt.^*}=\frac{1}{n}\frac{A_1\left(A_1+\mu^*A_{14}\right)}{\left(\mu^{*2}A_{14}+A_1\right)} \tag{31}$$

where $A_{14}=\frac{-A_5^2}{A_4}$.

The optimum values of μ, μ_1 and percent relative efficiencies E_1 and E_2 of the estimator *T* with respect to the estimator $\hat{M}_{y(n)}$ and Δ are computed for two natural populations and results are shown in Tabe-2, where

$$E_1=\frac{V\left(\hat{M}_{y(n)}\right)}{V\left(T\right)_{opt^*.}}\times 100 \text{ and } E_2=\frac{V\left(\Delta\right)_{opt^*.}}{V\left(T\right)_{opt^*.}}\times 100$$

7.1. Estimator Δ in practice

The main difficulty in using the proposed estimator Δ defined in equation (27) is the availability of k_5, as the optimum values of k_5 depends on the population parameters $P_{yx},\ f_y\left(M_y\right)$ and $f_x\left(M_x\right)$. If these parameters are known, the estimator Δ can easily be implemented, otherwise the unknown population parameters are replaced by their respective sample estimates as discussed in subsection 5.1. Hence, in this scenario the estimator Δ takes the following form:

$$\Delta^* = \psi_2 \Delta_1 + \left(1 - \psi_2\right)\Delta_2^* \tag{32}$$

where $\Delta_2^* = \left\{\hat{M}_{y(m)} + k_5^{**}\left(\hat{M}_{x(m)} - \hat{M}_{x(n)}\right)\right\}$, $k_5^{**} = \dfrac{-A_5^*}{A_4^*}$ and ψ_2 is the unknown constants to be determined so as to minimize the mean square error of the estimator Δ^*.

Remark 7.1.1. Since Δ^* is a regression-type estimator corresponding to the difference-type estimator Δ, hence up to the first order of approximations similar conclusions are applicable to Δ^* as that of Δ (See Singh and Priyanka (2008)).

Remark 7.1.2. For simulation study the proposed estimators T^* and Δ^* are considered instead of the proposed estimators *T* and Δ, respectively.

8. Monte Carlo Simulation

Empirical validation can be carried out by Monte Carlo Simulation. Real life situations of completely known two finite populations have been considered.

Population Source: [Free access to data by Statistical Abstracts of the United States]

The first population comprise *N = 51* states of the United States. Let y_i represent the number of abortions during 2007 in the i^{th} state of the US, x_i be the number of abortions during 2005 in the i^{th} state of the U,S and z_i denote the number of abortions during 2004 in the i^{th} state of the US. The data are presented in Figure 1.

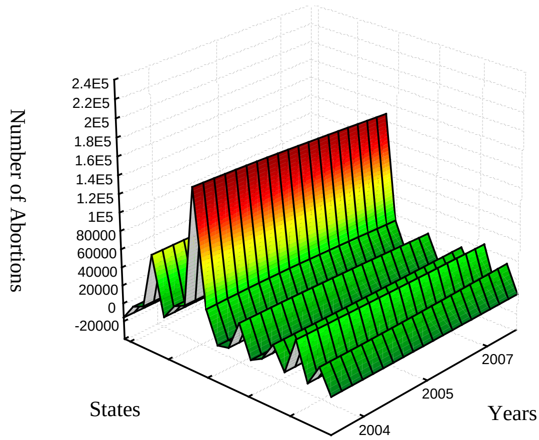

Figure 1. Number of abortions during 2004, 2005 and 2007 versus different states of the US

Similarly, the second population consists of $N=41$ corn producing states of the United States. We assume y_i the production of corn (in million bushels) during 2009 in the i^{th} state of the US, x_i be the production of corn (in million bushels) during 2008 in the i^{th} state of the US and z_i denote the production of corn (in million bushels) during 2007 in the i^{th} state of the US. The data are represented by means of graph in Figure 2.

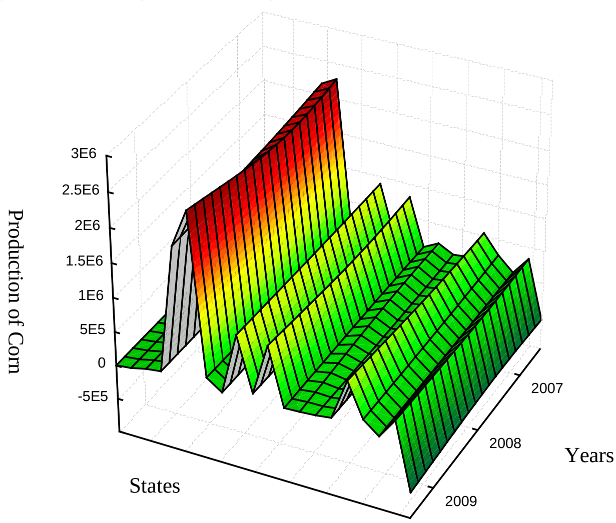

Figure 2. Production of corn during 2007, 2008 and 2009 versus different states of the US

The graphs in Figure1 and Figure 2 show that the number of abortions and the production of corn in different states are skewed towards right. One reason of skewness for the population-I may be the distribution of population in different states, that is the states having larger population are expected to have larger number of abortion cases. Similarly, for population-II the states having larger area for farming are expected to have larger production of corn. Thus, skewness of data indicates that the use of median may be a better measure of central location than mean in these situations.

For performing the Monte Carlo Simulation in the considered population-I, 5000 samples of *n=20* states were selected using simple random sampling without replacement in the year 2005. The sample medians $\hat{M}_{x(n)|k}$ and $\hat{M}_{z(n)|k}$, k =1, 2,---,5000 were computed and the parameters $f_x\left(M_x\right)$, $f_z\left(M_z\right)$ and P_{xz} were estimated by the method given in Remark 5.1.1. From each one of the selected samples, *m=17* states were retained and new *u=3* states were selected out of *N – n =51 – 20 = 31* states using simple random sampling without replacement in the year 2007. From the m units retained in the sample at the current occasion, the sample medians $\hat{M}_{x(m)|k}$, $\hat{M}_{y(m)|k}$ and $\hat{M}_{z(m)|k}$, *k = 1, 2,- - -,5000* were computed and the parameters $f_y\left(M_y\right)$, P_{yz} and P_{xz} were estimated. From the new unmatched units selected on the current occasion the sample medians $\hat{M}_{y(u)|k}$ and $\hat{M}_{z(u)|k}$, *k = 1, 2,- - -,5000* were computed. The parameters ψ_1 and ψ_2 are selected between 0.1 and 0.9 with a step of 0.1.

The percent relative efficiencies of the proposed estimator T^* with respect to $\hat{M}_{y(n)}$ and Δ^* are respectively given by:

$$E_{1sim} = \frac{\sum_{k=1}^{5000}\left[\hat{M}_{y(n)|k} - M_y\right]^2}{\sum_{k=1}^{5000}\left[T_k^* - M_y\right]^2}\times 100 \quad \text{and} \quad E_{2sim} = \frac{\sum_{k=1}^{5000}\left[\Delta_k^* - M_y\right]^2}{\sum_{k=1}^{5000}\left[T_k^* - M_y\right]^2}\times 100$$

For better analysis, this simulation experiments were repeated for different choices of μ.

Similar steps are also followed for Population-II. The simulation results in Table 3, Table 4 and Table 5 show the comparison of the proposed estimator T^* with respect to the estimators $\hat{M}_{y(n)}$ and Δ^*, respectively. For convenience the different choices of μ are considered as different sets for the considered Population-I and Population-II, which are shown below:

Sets	Population-I	Population-II
I	$n = 20;\ \mu = 0.15\ (m = 17,\ u = 3)$	$n = 15;\ \mu = 0.13\ (m = 13,\ u = 2)$
II	$n = 20;\ \mu = 0.25\ (m = 15,\ u = 5)$	$n = 15;\ \mu = 0.20\ (m = 12,\ u = 3)$
III	$n = 20;\ \mu = 0.35\ (m = 13,\ u = 7)$	$n = 15;\ \mu = 0.30\ (m = 10,\ u = 5)$
IV	$n = 20;\ \mu = 0.50\ (m = 10,\ u = 10)$	$n = 15;\ \mu = 0.40\ (m = 9,\ u = 6)$

Table 1. Descriptive statistics for Population-I and Population-II

	Population-I			**Population-II**		
	Abortions 2004 (z)	**Abortions 2005 (x)**	**Abortions 2007 (y)**	**Production of Corn in 2007 (z)**	**Production of Corn in 2008 (x)**	**Production of Corn in 2009 (y)**
Mean	23963.14	23651.76	23697.65	317997	294918.2	319313.7
Median	11010.00	10410.00	9600.00	83740	66650	79730
Standard Deviation	38894.81	38487.71	39354.65	565641.6	530483.7	563103.3
Kurtosis	12.02669	12.39229	14.42803	6.838888	6.492807	6.036604
Skewness	3.275197	3.310767	3.527683	2.638611	2.595704	2.499771
Minimum	80	70	90	2997	2475	2635
Maximum	208180	208430	223180	2376900	2188800	2420600
Count	51	51	51	41	41	41

Table 2. Comparison of the proposed estimator T (at optimal conditions) with respect to the estimators $\hat{M}_{y(n)}$ and Δ (at optimal conditions)

	Population - I	**Population-II**
μ_0	0.5411	0.6669
μ^*	0.6800	0.7642
E_1	1407.5	1401.3
E_2	1034.9	916.80

Table 3. Monte Carlo Simulation results when the proposed estimator T^* is compared to $\hat{M}_{y(n)}$ for Population-I and Population-II

	Population-I				Population-II			
Set	**I**	**II**	**III**	**IV**	**I**	**II**	**III**	**IV**
$\psi_1 \downarrow$	E_{1sim}	E_{1sim}	E_{1sim}	E_{1sim}	E_{1sim}	E_{1sim}	E_{1sim}	E_{1sim}
0.1	338.42	285.75	294.74	191.46	762.21	747.03	127.19	321.48
0.2	330.71	291.82	320.22	238.4	860.29	644.25	140.93	364.51
0.3	315.85	288.81	333.44	254.30	971.34	536.15	154.84	397.27
0.4	282.71	288.70	326.08	276.75	1097.6	427.33	166.51	420.99
0.5	248.64	268.90	322.70	295.47	1219.7	340.46	172.53	413.40
0.6	210.41	249.90	299.55	301.46	1377.0	262.76	175.98	413.49
0.7	178.81	220.94	269.87	304.12	1529.3	206.40	172.93	398.24
0.8	152.05	194.11	245.61	297.46	1707.7	166.72	166.51	369.96
0.9	127.19	168.82	216.58	289.94	1855.9	136.86	161.50	336.32

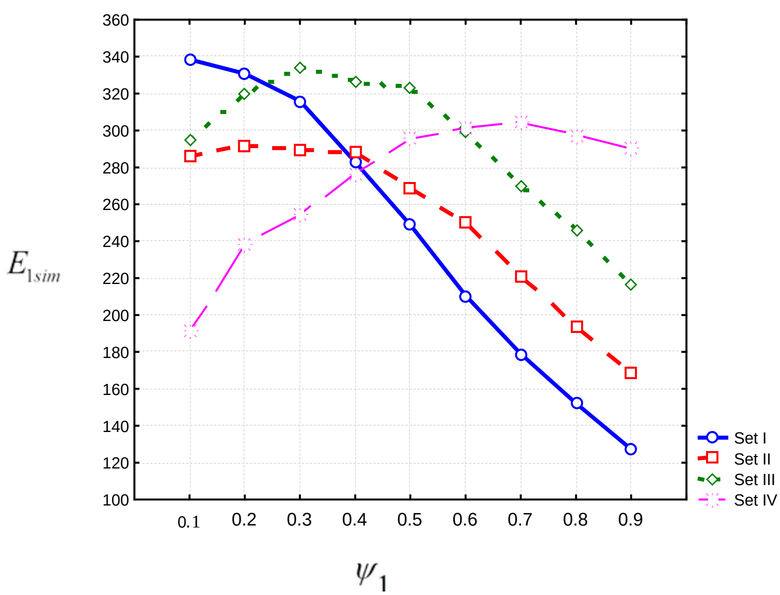

Figure 3. PRE of the estimator T^* with respect to $\hat{M}_{y(n)}$ for Population-I

Table 4. Monte Carlo Simulation results for Population-I when the proposed estimator T^* is compared to Δ^*

$\psi_1 \downarrow$	$\psi_2 \rightarrow$		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	E_{2sim}	I	329.1	470.4	707.2	1017.2	1590.3	2211.0	2869.2	4255.0	5490.3
		II	269.4	272.6	291.4	424.8	681.0	752.7	1023.3	1511.8	1790.9
		III	285.6	233.2	273.0	320.1	430.9	624.4	770.1	1126.7	1353.6
		IV	205.2	188.5	168.7	168.4	198.1	230.3	318.0	419.5	559.2
0.2	E_{2sim}	I	340.3	456.3	714.2	1078.2	1685.3	2268.1	3064.6	4227.3	5437.1
		II	285.8	282.7	312.6	461.3	678.1	824.9	1150.8	1600.8	2034.9
		III	295.9	251.1	279.7	344.3	457.5	636.8	831.4	1126.8	1428.8
		IV	242.3	199.2	177.2	182.9	222.9	269.7	351.5	483.4	631.6
0.3	E_{2sim}	I	325.9	440.9	688.6	1071.6	1547.1	2158.4	2979.3	4060.1	5145.1
		II	288.6	285.4	336.3	475.3	677.2	839.5	1187.6	1643.4	1983.4
		III	298.7	264.8	287.5	358.9	456.2	642.1	852.9	1159.3	1466.2
		IV	261.4	216.4	192.2	198.1	247.3	294.9	391.5	529.6	681.6
0.4	E_{2sim}	I	298.2	411.3	624.7	967.3	1430.2	1975.9	2648.7	3594.8	4721.6
		II	284.9	282.3	329.8	454.1	659.4	842.4	1152.1	1600.3	1946.5
		III	289.6	265.6	284.4	341.2	460.3	635.6	857.8	1142.6	1440.9
		IV	279.6	231.6	204.9	212.9	263.5	314.2	419.5	559.7	739.3
0.5	E_{2sim}	I	262.6	358.2	548.2	883.8	1247.1	1709.9	2238.4	3128.2	4213.1
		II	266.7	263.7	312.7	430.3	620.7	789.8	1072.8	1468.6	1775.0
		III	274.8	251.4	270.1	327.9	442.0	616.1	820.8	1111.1	1404.6
		IV	296.9	246.8	219.2	222.8	273.9	331.8	440.8	586.7	765.7
0.6	E_{2sim}	I	230.1	310.8	463.6	754.2	1078.0	1509.3	2016.2	2669.3	3583.8
		II	248.8	244.8	283.3	403.9	565.8	730.9	1004.8	1336.5	1673.8
		III	249.3	238.5	253.4	314.6	412.2	574.3	775.3	1016.9	1336.2
		IV	303.9	256.0	226.1	231.7	283.7	343.1	456.8	600.3	783.1
0.7	E_{2sim}	I	194.5	257.1	396.7	625.2	920.4	1275.6	1753.0	2249.7	2955.3
		II	226.0	216.7	252.9	352.7	512.4	656.3	907.6	1182.0	1473.9
		III	226.1	214.6	226.1	285.9	382.3	532.1	706.8	898.9	1208.2
		IV	305.8	258.3	227.1	235.5	284.2	346.9	459.8	599.8	788.4
0.8	E_{2sim}	I	159.8	221.7	341.1	523.4	757.4	1095.9	1515.0	1960.0	2478.9
		II	193.4	190.9	228.7	320.2	438.1	580.6	825.6	1037.5	1328.2
		III	201.6	194.7	205.2	265.1	347.7	481.8	628.9	800.2	1082.0
		IV	299.9	256.9	223.5	233.7	283.7	341.6	453.7	589.5	772.5
0.9	E_{2sim}	I	136.5	186.4	289.7	440.6	635.9	939.3	1269.8	1663.2	2125.0
		II	172.9	165.9	202.6	288.7	373.1	514.3	709.8	894.3	1160.4
		III	182.2	167.1	185.0	234.8	309.8	418.6	552.9	722.3	930.8
		IV	293.8	245.8	216.8	225.3	272.8	329.7	438.3	574.2	742.7

Table 5. Monte Carlo Simulation results for Population-II when the proposed estimator T^* is compared to Δ^*

$\psi_1 \downarrow$	$\psi_2 \rightarrow$		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	E_{2sim}	I	1126.40	2860.5	5849.0	9978.9	14402.0	22607.0	30230.0	40853.0	46469.0
		II	961.19	1757.9	3077.6	5323.8	7930.8	11637.0	14805.0	20847.0	26905.0
		III	274.83	264.72	298.76	362.76	515.77	742.68	1006.7	1174.6	1320.8
		IV	448.87	445.82	537.81	641.19	1000.5	1320.8	1757.2	2256.2	3038.8
0.2	E_{2sim}	I	873.59	2198.3	4489.6	7729.9	11800.0	17466.0	22954.0	31590.0	3644.3
		II	831.99	1472.2	2545.2	4305.6	6678.7	9960.1	13156.0	17250.0	23024.0
		III	302.79	284.98	314.11	406.01	562.11	821.52	995.42	1259.0	1522.1
		IV	495.59	481.24	567.79	708.65	1010.5	1426.0	1852.1	2354.0	3098.0
0.3	E_{2sim}	I	621.89	1594.20	3184.1	5627.4	8573.0	12582.0	16513.0	22385.0	27277.0
		II	682.77	1169.0	2044.1	3405.3	5386.4	7770.3	10373.0	13378.0	17978.0
		III	328.74	312.90	338.97	448.28	617.43	89.51	1079.6	1333.3	1719.8
		IV	528.81	521.64	667.01	761.28	1069.9	1502.1	1953.7	2645.4	3251.4
0.4	E_{2sim}	I	441.33	1136.90	2342.9	4039.8	6230.6	8970.8	11971.0	16010.0	20221.0
		II	540.36	905.32	1585.1	2637.0	4066.8	5938.0	8098.8	10354.0	13708.0
		III	349.27	334.32	366.96	469.80	658.16	909.27	1131.5	1455.1	1817.1
		IV	557.80	535.90	625.09	792.63	1111.7	1534.2	2022.3	2703.7	3360.2
0.5	E_{2sim}	I	325.32	829.35	1693.8	2954.8	4550.0	6503.2	8647.7	11725.0	14875.0
		II	423.09	685.55	1205.1	2062.0	3128.3	4491.7	6008.1	7843.8	10477.0
		III	358.42	347.77	382.11	498.04	683.40	938.99	1172.6	1524.7	1908.0
		IV	552.30	537.56	627.89	796.60	1104.7	1536.0	2036.20	2690.1	3371.6
0.6	E_{2sim}	I	247.94	628.85	1282.4	2233.8	3406.2	4921.7	6612.4	8869.5	11284.0
		II	326.45	531.46	954.37	1614.8	2416.2	3449.1	4720.8	6152.4	8021.9
		III	369.80	356.29	390.36	507.65	697.08	953.09	1193.9	1553.5	1966.7
		IV	545.08	519.34	607.57	778.51	1081.1	1486.7	1976.3	2607.6	3256.7
0.7	E_{2sim}	I	191.82	481.70	989.78	1738.2	2659.8	3832.4	5161.5	6844.7	8705.7
		II	256.24	421.16	747.44	1246.6	1864.4	2796.1	3789.1	4836.2	6404.1
		III	368.09	357.34	391.04	507.07	692.18	943.99	1198.0	1548.7	1972.1
		IV	523.74	448.94	569.41	738.38	1020.9	1405.1	1886.9	2452.8	3067.3
0.8	E_{2sim}	I	154.29	383.89	790.48	1385.5	2112.4	3041.20	4114.9	5376.9	6949.5
		II	206.36	335.56	604.62	1004.1	1507.5	2283.7	3062.3	3868.2	5119.9
		III	361.45	347.49	391.04	490.64	667.61	915.93	1161.0	1510.2	1915.8
		IV	488.89	463.14	526.20	689.27	941.81	1304.0	1735.1	2254.4	2837.2
0.9	E_{2sim}	I	124.89	310.43	635.21	1100.2	1714.1	2458.4	3302.5	4362.3	5601.2
		II	169.07	271.88	498.12	826.69	1245.4	1855.6	2493.5	3169.4	4211.6
		III	346.69	330.68	379.63	469.72	629.28	869.77	1114.2	1438.0	1843.1
		IV	445.87	413.45	477.73	615.16	848.82	1179.9	1569.1	2032.7	2622.9

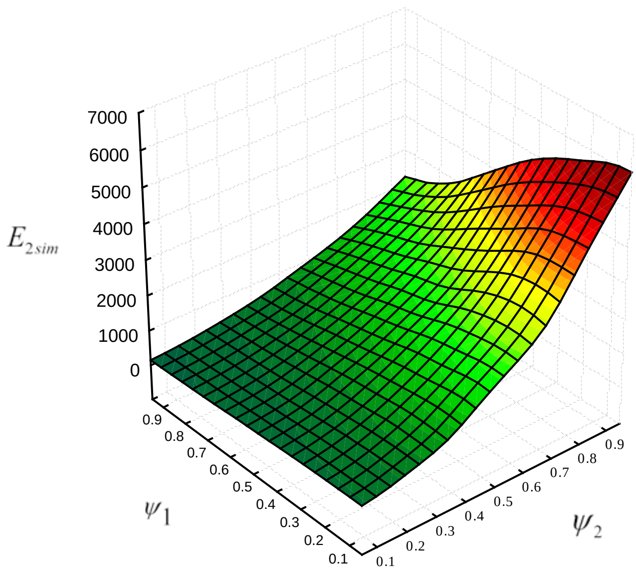

Figure 4. PRE of estimator T^* with respect to Δ^* for set-I for Population-I

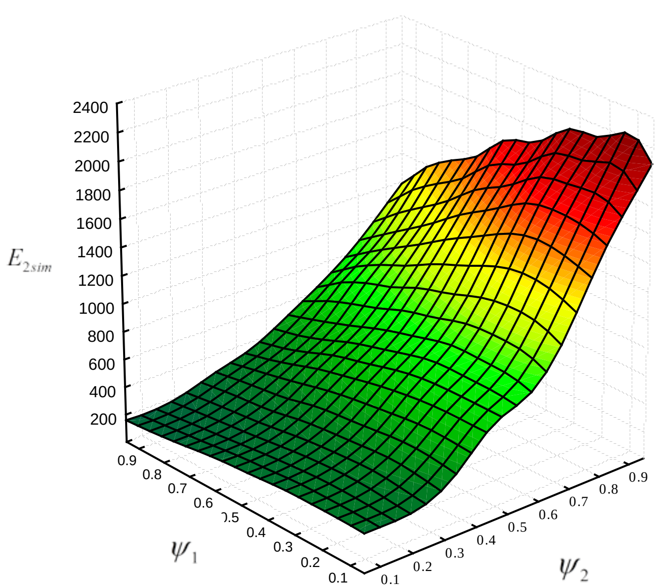

Figure 5. PRE of estimator T^* with respect to ψ_1 for set-II for Population-I

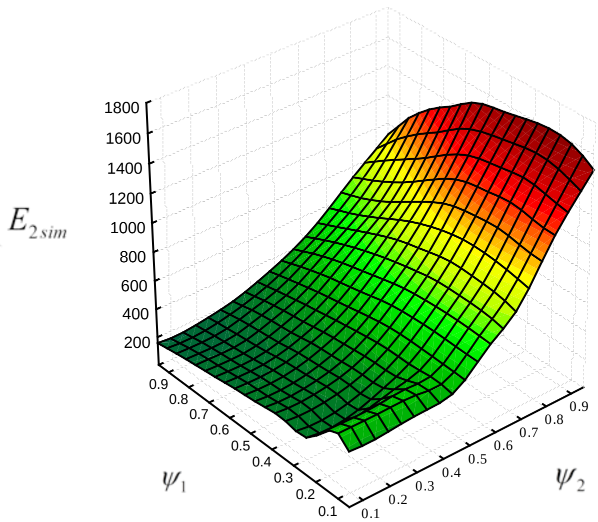

Figure 6. PRE of estimator T^* with respect to ψ_1 for set-III for Population-I

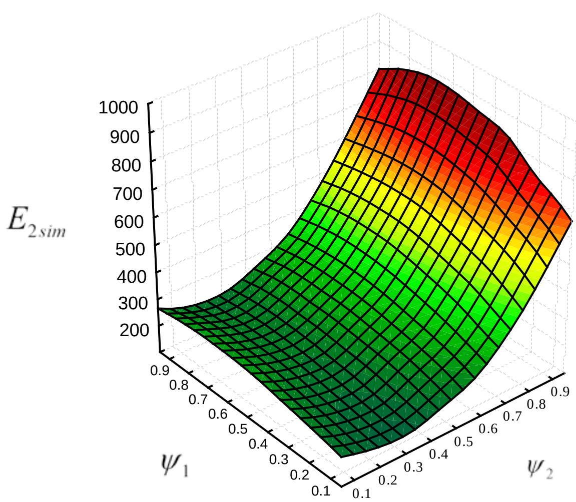

Figure 7. PRE of estimator T^* with respect to ψ_1 for set-IV for Population-I

9. Analysis of empirical and simulation results

1. From table 2 it is visible that the optimum values of μ (fraction of a fresh sample to be drawn at current occasion) exist and this value for the estimator T is less than that of the estimator Δ for both the considered populations. This indicates that the use of additional auxiliary information at both the occasion reduces the cost of the survey.

2. Appreciable gain is observed in terms of precision indicating the proposed estimator T (at optimal condition) preferable over the estimators $\hat{M}_{y(n)}$ and Δ (at optimal condition). This result justifies the use of additional auxiliary information at both the occasions in two-occasion successive sampling.

3. The following conclusion may be observed from Table 3 and Figure 3:

   (i) For Set-I of Population-I, the value of E_{1sim} decreases as the value of ψ_1 increases. This result is expected as for Set-I the value of μ is very low, however for Set-I of Population-II E_{1sim} increases with the increasing value of ψ_1.

   (ii) For Set-II, III and IV of the Population-I, the value of E_{1sim} first increases and then starts decreasing with the increasing value of ψ_1, however no specific pattern is observed for set II, III and IV of Population-II.

   (iii) For all the considered combinations appreciable gain in precision is observed when the proposed estimator is compared with the sample median estimator. Hence, the use of additional auxiliary information at both the occasions is highly justified.

4. The following points may be noted from Table 4, Table 5 and Figures 4, 5, 6 and 7:

   (i) For fixed value of ψ_1 and ψ_2, the value of E_{2sim} decreases with the increasing value of μ, except for few combinations of ψ_1 and ψ_2 for Population-I, however no specific pattern is observed for Population-II.

   (ii) For fixed value of ψ_1 and μ and increasing value of ψ_2, the value of E_{2sim} also increases, except for few combinations.

   (iii) For fixed value of ψ_2, and lower value of μ, the value of E_{2sim} decreases with increasing value of ψ_1, however for higher value of μ, the value of E_{2sim} increases with the increasing value of ψ_1, except for few combinations.

   (iv) Tremendous gain in precision is obtained for all the considered cases.

10. Conclusion

From the analysis of empirical and simulation results it can be concluded that the proposed estimator T compares favourably in terms of efficiency with the standard sample median estimator, where there is no matching from previous occasion. The estimator T also proves to be much better than the estimator Δ, when no additional auxiliary information is used at any occasion. Therefore, the use of additional auxiliary information at both the occasions in two occasion successive sampling for estimating population median at current occasion is highly rewarding in terms of precision and reducing the total cost of survey. Hence, the proposed estimators may be recommended for further use by survey practitioners.

Acknowledgements

Authors are thankful to the anonymous referees for their precious and constructive suggestions regarding improvement of the article. Authors appreciate the free access and acknowledge the use of data from Statistical Abstracts of the United States, 2012 that is available on the Internet. Authors are also thankful to UGC, New Delhi, India for providing the financial assistance to carry out the present work.

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